Chapter 4

Spatial Market Equilibria on Networks†

Within the framework of bilevel facility location models in spatial economic competition, there are several different submodels that one can employ to describe economic competition and determine a market equilibrium among firms on the network. Such economic models include the classic spatial price equilibrium (SPE) model, first introduced by Samuelson (1952), and enhanced for over three decades since then (e.g., Takayama and Judge, 1964) as well as models of oligopolistic competition (e.g., Cournot-Nash). In this chapter, we will review the classic SPE model and the oligopolistic spatial Cournot-Nash model. Additionally, we will review non-extremal versions of these models (i.e., variational inequality formulations) and consider their existence and uniqueness properties.

4.1 The Spatial Price Equilibrium Model

The spatial price equilibrium (SPE) model (Samuelson, 1952 and Takayama and Judge, 1964 and 1971) has remained a widely-used modeling approach for describing an economic equilibrium for a commodity.

†Some of the material in this chapter is adapted and expanded from Friesz, Miller and Tobin (1988A), from Friesz, Tobin and Miller (1989) and from Miller, Tobin and Friesz (1991).
among spatially separated firms and consumers. The SPE model typically poses a supply and a demand function for a commodity at each spatially separated market or node over a defined network. Additionally, it specifies the arc (or path) functions describing the interregional transportation costs (and infrastructure) for commodity movements between markets. Briefly, a solution to an SPE model obtains a vector of equilibrium or market clearing prices for a commodity at each node of the network, as well as a vector of "economically rational" commodity flows between these markets. This model has, in fact, formed the basis for numerous empirical studies of the flow of good(s) among spatially separated regions (e.g., Takayama and Judge, 1973). The SPE's relative ease of solution as an "equivalent" convex nonlinear mathematical program with linear constraints (Tobin and Friesz, 1983) has further enhanced its appeal.

Perhaps the major theoretical difficulty in utilizing the SPE model to analyze or predict real world economic equilibria emanates from the strong assumptions underlying this model. In particular, the SPE model assumes both perfect competition for producers and consumers at each node or region of the network, and perfect competition among the transportation agents providing freight services between these regions. This implies commodity prices set equal to the marginal costs of producing firms in each region, as well as commodity freight rates set equal to the transportation firms' marginal costs.

Harker (1986) notes the unrealistic restrictions of these assumptions for problems with a discrete spatial dimension, and therefore, proposes several variations upon the SPE model which allow for imperfect competition. As evidence of the need for these models, he cites the well known existence of spatial monopolies which can result from the fact that a firm closer to a market will have an advantage over a firm further away, given that the service of transportation has a cost. In particular, Harker formulates two spatial monopoly models and a Cournot-Nash oligopoly model. As a foundation for deriving each of these models, Harker creates a revised SPE model. In this revised formulation, he obtains the supply function as the solution to a cost minimization problem, rather than explicitly stating the supply function as in the standard SPE model.