7 Perturbations in Beam Dynamics

The study of beam dynamics under ideal conditions is the first basic step toward the design of a beam transport system. In the previous sections we have followed this path and have allowed only the particle energy to deviate from its ideal value. In a real particle beam line or accelerator we may, however, not assume ideal and linear conditions. More sophisticated beam transport systems require the incorporation of nonlinear sextupole fields to correct for chromatic aberrations. Deviations from the desired field configurations can be caused by transverse or longitudinal misplacements of magnets with respect to the ideal beam path. Of similar concern are errors in the magnetic field strength, undesirable field effects caused in the field configurations at magnet ends, or higher order multipole fields resulting from design, construction, and assembly tolerances. Still other sources of errors may be beam-beam perturbations, insertion devices in beam transport systems or accelerating sections which are not part of the magnetic lattice configurations. Such systems may be magnetic detectors for high energy physics experiments, wiggler and undulator magnets for the production of synchrotron radiation, a gas jet or immaterial field sources like that of a free electron laser interacting with the particle beam to name just a few examples. The impact of such errors is magnified in strong focusing beam transport systems as has been recognized soon after the invention of the strong focusing principle. Early overviews and references can be found for example in [7.1–12].

A horizontal bending magnet has been characterized as a magnet with only a vertical field component. This is true as long as this magnet is perfectly aligned, in most cases perfectly level. Small rotations about the magnet axis result in the appearance of horizontal field components which must be taken into account for beam stability calculations.

We also assumed that the magnetic field in a quadrupole vanishes at the center of magnet axis. In the horizontal midplane of a quadrupole the vertical field component has been derived as $B_y = g x$. If this quadrupole is displaced horizontally with respect to the beam axis by a small amount $\delta x$ we observe a dipole field $\delta B_y = g \delta x$ at the beam axis. Similarly, a horizontal dipole field component is created for a vertical displacement of the quadrupole. These dipole field components in most cases are unintentional and lead to an undesired deflection of the beam.
In addition, a quadrupole can be rotated by a small angle with respect to the reference coordinate system. As a result we observe the appearance of a small component of a "rotated quadrupole". A sextupole magnet, when displaced, introduces a dipole as well as a quadrupole field component on the beam axis. In general we find that any displaced higher order multipole introduces field errors on the beam axis in all lower order field configurations.

Although such misalignments and field errors are unintentional and undesired, we have to deal with their existence since there is no way to avoid such errors in a real environment. The particular effects of different types of errors on beam stability will be discussed. Tolerance limits on these errors as well as corrective measures must be established to avoid destruction of the particle beam. Common to all these perturbations from ideal conditions is that they can be considered small compared to forces of linear elements. We will therefore discuss mathematical perturbation methods that allow us to determine the effects of perturbations and to apply corrective measures for beam stability.

7.1 Magnet Alignment Errors

In this section field errors created by magnet misalignments like displacements or rotations from the ideal location will be derived quantitatively. Such magnet alignment errors, however, are not the only cause for field errors. External sources like the earth magnetic field, the fields of nearby electrical current carrying conductors, magnets connected to vacuum pumps or ferromagnetic material in the vicinity of beam transport magnets can cause similar field errors. For example electrical power cables connected to other magnets along the beam transport line can be hooked up such that the currents in all cables are compensated. This occurs automatically for cases, where the power cables to and from a magnet run close together. In circular accelerators one might, however, be tempted to run the cables around the ring only once to save the high material and installation costs. This, however, causes an uncompensated magnetic field in the vicinity of cables which may reach as far as the particle beam pipe. The economic solution is to seek electrical current compensation among all magnet currents by running electrical currents in different directions around the ring. Careful design of the beam transport system can in most cases minimize the impact of such field perturbations while at the same time meeting economic goals.

Incidental field errors cannot be derived in a formal way but must be evaluated individually by magnetic measurements. The main component of such fields, however, can be described in most cases by a superposition of a dipole and a gradient field. In the following paragraphs we will restrict ourselves to the effects of magnet field and alignment errors. Misalignment errors can be expressed by the transformation