7. Random Choice and Related Methods

7.1 Introduction

In 1965, Glimm [141] introduced the Random Choice Method (RCM) as part of a constructive proof of existence of solutions to a class of non-linear systems of hyperbolic conservation laws. In 1976, Chorin [73] successfully implemented a modified version of the method, as a computational technique, to solve the Euler equations of Gas Dynamics. In essence, to implement the RCM one requires (i) exact solutions of local Riemann problems and (ii) a random sampling procedure to pick up states to be assigned to the next time level. As we shall see, there is a great deal of commonality between the RCM and the Godunov method presented in Chap. 6. Both schemes use the exact solution of the Riemann problem, although Godunov's method can also be implemented using approximate Riemann solvers, as we shall see in Chaps. 9 to 12. The two methods differ in the way the local Riemann problem solutions are utilised to march to the next time level: the Godunov method takes an integral average of local solutions of Riemann problems, while the RCM picks a single state, contained in the local solutions, at random. The random sampling procedure is carried out by employing a sequence of random numbers. The statistical properties of these random numbers have a significant effect on the accuracy of the Random Choice Method.

Since the introduction of the RCM as a computational scheme by Chorin, there have been many contributions to the development of the method. Chorin himself [74] extended the RCM to combustion problems; Sod [317] applied the RCM to the one-dimensional Euler equations for cylindrically and spherically symmetric flows, thereby introducing a way of dealing with algebraic source terms. Concus [92] applied the RCM to a non-linear scalar equation governing the two-phase flow of petroleum in underground reservoirs. Major contributions to the method were presented by Colella [85], [86]; these include a better understanding of the method, its strengths and limitations, and improved random sampling techniques. Marshall and Mendez [230] applied the RCM to the one-dimensional shallow water equations. Li and Holt [213] applied the RCM to the study of underwater explosions. Marshall and Plohr [231] applied the RCM to solve the steady supersonic Euler equations, see also Shi and Gottlieb [308], and to the study of shock wave diffraction phenomena. Gottlieb [149] compared the implementation of the
RCM on staggered and non–staggered grids and introduced an effective way of using irregular meshes. Toro [351] applied the RCM to covolume gases with moving boundaries. Applications of the RCM to the study of reactive flows were performed by Saito and Glass [293], Takano [337], Singh and Clarke [314] and Dawes [104]. Olivier and Gröning [248] applied the RCM to solve the two–dimensional time dependent Euler equations to study shock focussing and diffraction phenomena in water and air.

Essentially, the RCM is applicable to scalar problems in any number of dimensions and to non–linear systems in two independent variables. Examples of these systems are the one–dimensional, time dependent Euler equations, the two–dimensional, steady supersonic Euler equations and the one–dimensional shallow water equations. By using splitting schemes, see Chap. 15, one can also solve extensions of these systems to include algebraic source terms or even terms to model viscous diffusion; see Sod [320] and Honma and Glass [174]. A fundamental limitation of the RCM is its inability to solve multi–dimensional non–linear systems via splitting schemes, which usually work well when extending other schemes to multi–dimensional problems; see Chap. 16. An attraction of the RCM is its ability to handle complex wave interaction involving discontinuities such as shock waves and material interfaces; these are resolved as true discontinuities. Most other methods will smear discontinuities over several computing cells, a problem that is particularly serious for contact surfaces. Although computed discontinuities in the RCM have infinite resolution, the position of these waves at any given time has an error, which is random in character. The randomness of the RCM also shows in resolving smooth waves, such as rarefactions. Such randomness is tolerable when solving homogeneous systems, i.e. no source terms. In the presence of source terms however, the randomness tends to be enhanced.

This chapter is primarily devoted to the conventional Random Choice Method, but we also present what appears to be a new random choice method [365] that is analogous to the Lax–Friedrichs (deterministic) scheme. In addition we present a, deterministic, first–order centred (FORCE) scheme based on a reinterpretation of the conventional RCM on a staggered grid. The presentation of the schemes is given in terms of the time–dependent, one dimensional Euler equations. The reader is advised to review Chap. 4 before proceeding with the study of the present chapter.

### 7.2 RCM on a Non–Staggered Grid

We consider the general Initial Boundary Value Problem (IBVP) for non–linear systems of hyperbolic conservation laws, namely

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\begin{align*}
\text{PDEs} & : U_t + F(U)_x = 0 , \\
\text{ICs} & : U(x,0) = U^{(0)}(x) , \\
\text{BCs} & : U(0,t) = U_l(t) , U(L,t) = U_r(t) .
\end{align*}
\]

(7.1)