7 Collective Dynamics of Disc Particles

I Formalism

Bel-Affris: “When the wall came nigh, it changed into a line of men . . . . Every man of them flung his javelin . . . .”

G.B.Shaw, Caesar and Cleopatra, Act I.

7.1 Transport Theories for Macroparticles

One can understand the reasons why the brightness of the Saturnian A ring shows an azimuthal asymmetry by considering the dynamics of the separate particles. However, the layering of the Saturnian rings and the occurrence of other spatial structures in the planetary rings are caused by collective processes and it is natural to study those in the framework of a hydrodynamical model where the “gas” of the colliding macroparticles is described in the same way as an ordinary molecular gas. The results of Chaps. 4 and 5 show that one can take as a typical ring particle a practically completely inelastic loose meter-size sphere. One must here take into account the gravitational field of such particles; this plays an important role in collision processes and in the break-up of large bodies and the motion of the small fragments. It is impossible to speak about the applicability of hydrodynamics to planetary rings without indicating the characteristic sizes and time scales of the processes which are described; they must be significantly larger than the mean free path and the mean free flight time of a particle, respectively. We shall show in Chap. 8 that these inequalities are satisfied for the large-scale processes in which we are interested.

Let us introduce the fundamental concept of a particle distribution function. Each particle in the gas is characterised by three coordinates, \( x, y, z \), and three velocity components, \( v_x, v_y, v_z \). These six quantities, considered as coordinates form a six-dimensional space in which the gaseous medium occupies some volume. The particle density in the six-dimensional coordinate-velocity space is the particle distribution function \( f(r, v, t) \). Its evolution is described by the kinetic equation

\[
\frac{\partial f}{\partial t} + \left( v \cdot \frac{\partial f}{\partial r} \right) + \left( \frac{dv}{dt} \cdot \frac{\partial f}{\partial v} \right) = C(f), \quad \frac{dv}{dt} = \frac{F}{m},
\]

(7.1)

where \( F \) is the total force acting upon a particle and \( C(f) \) is the collisional term which describes the evolution of the distribution function due to collisi-
sions between the particles. The actual form of $F$ and $C(f)$ depends on the kind of system and its characteristics.

Once we know the distribution function $f(r, v, t)$ we can obtain the usual average characteristics of the gas:

1) the density,

$$n(r, t) = \int f(r, v, t) \, d^3v,$$  \hfill (7.2)

2) the average (hydrodynamic) velocity,

$$V(r, t) = \frac{1}{n} \int v \, f(r, v, t) \, d^3v,$$  \hfill (7.3)

3) and the temperature,

$$T(r, t) = \frac{1}{n} \int \frac{1}{3} m(v - V)^2 f(r, v, t) \, d^3v.$$  \hfill (7.4)

Hydrodynamics for rotating discs

Fig. 7.1. Algorithm of the method for constructing the hydrodynamic (moment) equations for a rotating flat system of inelastically colliding particles.

The classical methods of kinetic theory, developed by Chapman, Enskog, and other authors (Chapman and Cowling 1953, Hirschfelder et al. 1954, Braginskii 1958, 1965, Shkarovskii et al. 1969, Ferziger and Kaper 1972, Alekseev 1982), made it possible to obtain from the kinetic equation (7.1) a closed