9 Neutrino Mixing Schemes

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9.1 Introduction

A revolution in our understanding of the neutrino sector is under way, driven by observations that are interpreted in terms of changes in the flavors of neutrinos as they propagate. Since neutrino oscillations occur only if neutrinos are massive, these phenomena indicate physics beyond the Standard Model. With the present evidence for oscillations from atmospheric, solar and accelerator data, we are already able to begin to make strong inferences about the mass spectrum and the mixings of neutrinos. Theoretical efforts to achieve a synthesis have produced a variety of models with differing testable consequences. A combination of particle physics, nuclear physics and astrophysics is needed for a full determination of the fundamental properties of neutrinos. This chapter reviews what has been achieved thus far and the future prospects for understanding the nature of neutrino masses and mixing.

9.2 Two-Neutrino Analyses

In a model with two neutrinos, the probability of a given neutrino flavor $\nu_{\alpha}$ oscillating into $\nu_{\beta}$ in a vacuum is

$$P(\nu_{\alpha} \to \nu_{\beta}) = \sin^2 2\theta \sin^2 \left( 1.27 \frac{\delta m^2 L}{E} \right),$$

where $\theta$ is the two-neutrino mixing angle, $\delta m^2$ is the mass-squared difference of the two mass eigenstates in $\text{eV}^2$, $L$ is the distance from the neutrino source to the detector in kilometers and $E$ is the neutrino energy in GeV.

9.2.1 Atmospheric Neutrinos

The atmospheric neutrino experiments determine the ratios

$$\frac{N_{\mu}}{N_{\mu}^0} = \alpha \left[ \langle P(\nu_{\mu} \to \nu_{\mu}) \rangle + r \langle P(\nu_e \to \nu_{\mu}) \rangle \right],$$

$$\frac{N_{e}}{N_{e}^0} = \alpha \left[ \langle P(\nu_e \to \nu_e) \rangle + r^{-1} \langle P(\nu_{\mu} \to \nu_e) \rangle \right],$$

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where $N^0_e$ and $N^0_\mu$ are the expected numbers of atmospheric $e$ and $\mu$ events, respectively, in the absence of oscillations; $r = N^0_e/N^0_\mu$, $\langle \rangle$ indicates an average over the neutrino spectrum; and $\alpha$ is an overall neutrino flux normalization. Atmospheric neutrino data have generally indicated that the expected number of muons detected is suppressed relative to the expected number of electrons [9.1]; this suppression can be explained via neutrino oscillations [9.2].

The atmospheric data also indicate that $N_e/N^0_e$ is relatively flat with respect to zenith angle, while $N_\mu/N^0_\mu$ decreases with increasing zenith angle (i.e. with longer oscillation distance). Assuming $\nu_\mu \to \nu_e$ oscillations, the favored two-neutrino parameters are [9.3]

$$ \delta m^2 = 3.5 \times 10^{-3} \text{eV}^2 \quad 1.5 - 7 \times 10^{-3} \text{eV}^2, \quad (9.4) $$
$$ \sin^2 2\theta = 1.00 \quad 0.80 - 1.00; \quad (9.5) $$

the 90% C.L. allowed ranges are given in parentheses. The absolute normalization of the electron data indicates $\alpha \simeq 1.18$, which is within the theoretical uncertainties [9.4]. The flatness versus $L$ of the electron data implies that simple $\nu_\mu \to \nu_e$ oscillations are strongly disfavored. Large-amplitude ($\sin^2 2\theta > 0.2$) $\nu_\mu \to \nu_e$ oscillations are also excluded by the Chooz reactor data [9.5] for $\delta m^2_{\text{atm}} \gtrsim 10^{-3} \text{eV}^2$.

It is also possible that atmospheric $\nu_\mu$ are oscillating into sterile neutrinos. However, measurements of the upgoing zenith angle distribution and $\pi^0$ production disfavor this possibility ([9.6] and Chap. 5 of this book).

### 9.2.2 Solar Neutrinos

For the $^{37}\text{Cl}$ [9.7] and $^{71}\text{Ga}$ [9.8] experiments, the expected number of neutrino events is

$$ N = \int \sigma P(\nu_e \to \nu_e)(\beta \phi_B + \phi_{\text{non-B}}) \, dE, \quad (9.6) $$

where we allow an arbitrary normalization factor $\beta$ for the $^8\text{B}$ neutrino flux since its normalization is not certain. For the Kamiokande [9.9] and Super-Kamiokande [9.10] experiments the interaction is $\nu e \to \nu e$ and the outgoing electron energy is measured. The number of events per unit of electron energy is then

$$ \frac{dN}{dE_e} = \beta \int \left( \frac{d\sigma_{\text{CC}}}{dE'_e} P(\nu_e \to \nu_e) + \frac{d\sigma_{\text{NC}}}{dE'_e} [1 - P(\nu_e \to \nu_e)] \right) \times G(E'_e, E_e) \phi_B \, dE'_e \, dE_e, \quad (9.7) $$

where $d\sigma_{\text{CC}}/dE'_e$ and $d\sigma_{\text{NC}}/dE'_e$ are the charged-current and neutral-current differential cross sections, respectively, for an incident neutrino energy $E'_\nu$, and $G(E'_e, E_e)$ is the probability that an electron of energy $E'_e$ is measured as having energy $E_e$. The neutrino fluxes are taken from the standard solar model (SSM) [9.11]. If $\nu_e$ oscillates into a sterile neutrino, $\sigma_{\text{NC}} = 0$. 