3 ROLE OF FEM IN SPRING ANALYSIS

Generally speaking, the analytical approach for spring calculations has been fully developed. Further effort to bring about new calculation equations would not result in additional benefit. Therefore, it seems that only numerical analysis is left to us to improve the accuracy. There are many levels in the conventional analysis technique, and the necessity for selecting the appropriate formula is attributed to the limited area of application of each formula. We will examine the case of a helical compression spring.

The design formula of the helical spring (3.1) has been given in many handbooks and should be familiar to our readers. These formulas are constructed on an assumption that a model of one-degree of freedom replaces the actual spring as an elastic component described in chapter 1. In this case, no degree of freedom is allowed in the transversal direction and we cannot get the transverse stiffness (κ in subsection 1.4.5). Now, what is the usual way for finding the transversal stiffness of actual springs? The makeshift formula for the transversal stiffness[1] could be used. This formula is nothing more than an elastica, which is a model with an increased degree of freedom in an elastic component. It naturally has a limit in its application. For example, it is natural that the transversal stiffness has its own direction that originates from relative spring end position and the direction of transverse force. But, it is fundamentally impossible to consider the direction of the transversal stiffness as long as the spring is treated as an elastica.

When we want to analyze such details of the spring, the only method is to formulate the spring as a curved beam with three-dimensional six-degrees of freedom. There are several such outcomes[2, 3]. While it is possible to analyze the direction of the transversal stiffness or the eccentricity of force using these formulas, the problem is the laborious calculation. While solving can be carried out by hand for simple formulas, computer is essential to make calculation on much complicated formulas.

Although the formula appears to be the best way for analysis, it has its own limits due to its analytical character. That is, the shape of the spring must be expressed mathematically; meaning the shape of the spring is limited to something fairly primitive. The suspension spring for passenger cars, for instance, has a very complicated shape. It might be conically wound on the ends in some cases. It might even be tapered along the length of the wire. The conventional calculation cannot cope with such a complicated shape. Then, the conception of FEM of dividing the spring into elements was inevitably formulated[4]. In the early days, however, we did nothing but make the computer program ourselves.
This situation was suddenly changed when the commercial FEM program became popular. Even if we cannot make a computer program, it is possible to carry out computations with proper sequence of data input, when the FEM software is used as a black box. The popularization of such techniques has liberated the engineer from laborious calculations.

In reviewing the history of analysis for the helical spring, the reader might have noticed that the analytical technique has arrived at the final stage of its improvement. Looking at springs of other types, we unfortunately find many springs that are still at the level of hand calculation and have not benefited from recent improvements in analysis. Flat springs and formed wire springs have been left almost untouched. This cannot be attributed, however, to the negligence of the engineer. These springs are so diversified in their shapes, that even a simplified equation could not be formulated thus far. There was nothing else to do but apply Castigliano’s theorem to each case and to establish an appropriate design formula for the particular shape. For springs of these types, the appearance of FEM beyond doubt, an absolute lifesaver. FEM does not need the design formula, and shows its proper function in an a-la-carte menu computation.

What the reader may want to know might not be the content of FEM, but its usefulness in daily works. It is the main objective of this chapter to show the merit of FEM for spring technique. In a sense, this is a kind of tour to visit the greenroom of spring theater.

### 3.1 COMPARISON OF FEM WITH CONVENTIONAL DESIGN METHODS

#### 3.1.1 Assumption in Model Construction

(1) **Simplified formula for helical springs**  The most popular calculating formula of springs are those for cylindrical compression springs with circular cross-sectional wire. With following notation,

- \( k \) = spring constant,
- \( d \) = wire diameter,
- \( D \) = mean coil diameter,
- \( n \) = effective number of coils,
- \( G \) = shear modulus,
- \( \tau \) = shear stress (modified),
- \( \kappa \) = curvature correction factor,

the formulas are given by

\[
\text{spring constant: } k = \frac{Gd^4}{8nD^3} , \tag{3.1}
\]

\[
\text{surface stress: } \tau = \kappa \frac{8PD}{\pi d^3} . \tag{3.2}
\]

Equation (3.1) specifies spring characteristic, and (3.2) indicates spring strength.

The assumptions used in the derivation of (3.1) are: