The pattern of rotating spiral waves is typical for excitable media. Such waves are observed in chemical reactions in aqueous solutions [521,522], in catalytic chemical reactions on metal surfaces [523], in liquid crystals [524], in optical systems [525], in populations of the social amoebae *Dictyostelium discoideum* [526,527], in the heart [528], in the retina [115], and even inside single biological cells [529]. Remarkably, the properties of spiral waves in all these diverse systems are similar. This suggests that the emergence and the dynamics of spiral waves are based on general mechanisms, not sensitive to the details of a particular system.

The mathematical theory of pattern formation in excitable media originates from the pioneering work by Norbert Wiener. The great mathematician of the 20th century, the founder of cybernetics, was striving to understand the processes of self-regulation in living matter. During a stay of Wiener in Mexico, he had many discussions with his friend Arturo Rosenblueth from the Mexican Institute of Cardiology. They talked about patterns of circulating electrical excitations in the heart, their similarity to the processes in the nervous tissue and mathematical models able to reproduce this behavior. These discussions and subsequent studies resulted in their article “The mathematical formulation of the problem of conduction of impulses in a network of connected excitable elements, specifically in cardiac muscle” [530] which appeared in 1946 in *Archivos del Instituto de Cardiología de México*. In this large paper Wiener and Rosenblueth formulated the basic mathematical concept of an excitable medium, showed the existence of rotating spiral waves, analyzed their properties and considered the role of such patterns in cardiac disorders and fibrillation.

The paper was actively discussed in the seminars at the Massachusetts Institute of Technology (B. Hess, personal communication). It was followed by publications [531,532] where its generalizations and more detailed studies were performed. My own copy of this article stems from A.T. Winfree and contains numerous comments made by him on the margins, as he was reading the paper in 1970 at the University of Chicago. In the 1960s this rare publication was discovered by the participants of the Seminar on Mathematical Biology, held by I.M. Gel’fand at the Mathematical Department of the Moscow University, and its Russian translation has been published.

Modern theoretical studies of pattern formation in excitable media are much influenced by the subsequent discovery of partial differential equations...
that describe reaction-diffusion systems with excitable properties. The highly increased computation speed permits fast numerical simulations of such systems. However, in many biological applications such detailed equations are not known and their very existence is questionable. In contrast to this, the more abstract approach, advocated by N. Wiener, has wider applicability. In its framework, an excitable medium is defined by a number of its characteristic parameters, such as the wave propagation velocity, the excitation and recovery times, etc. These parameters can often be directly deduced from the experiments. Once they are known, the theory predicts the properties of spiral waves and their dynamics.

The aim of this article is to review the impact of the work by Wiener and Rosenblueth on modeling of pattern formation in excitable media. First I summarize the contents of their fundamental paper and then look at how this approach has been extended to describe various phenomena related to spiral waves in excitable media.

9.1 Excitable Media

The Wiener–Rosenblueth (WR) theory of excitable media was based on the experiments with cardiac and nervous tissues performed in the beginning of the 20th century [533–536]. In these early publications, such concepts as an excitation wave or the refractory period were introduced. However the description of excitable media by Wiener and Rosenblueth was much more general and also included references to physical aspects of these phenomena, as can be seen from the following citations:

"Conduction in nervous tissue resembles that in somatic striated in cardiac muscle. The laws which apply to the muscle fibers are also applicable to the nerve fibers. In both instances the propagation is active, with energy supplied locally. In both cases an impulse travels with a nearly uniform velocity. In both cases the excitation and transmission are all-or-none and do not allow for impulses of varying degrees of strength. In both cases activity is followed by an unexcitable period of definite duration, the absolutely refractory period; and this stage is followed in turn by a relatively refractory period, during which the tissues have subnormal excitability" ([530], p. 205–206).

The structure of the cardiac muscle is synticial. Its fibers interconnect abundantly and excitation can easily pass through any connection. Therefore, the cardiac muscle can be treated as a pseudo-continuous conductive medium. In contrast to this, conduction in nervous structures is more complicated (see, for example, Chap. 2). When an impulse reaches a synapse, it does not necessarily stimulate the following neuron. Transsynaptic stimulation usually requires a repetition of impulses in time at the same synapses (temporal summation) or the simultaneous arrival of impulses at a sufficient number of adjacent synapses. Moreover, the arrival of impulses may have other consequences than the stimulation of the postsynaptic element. It may