Considering the position of a sample as random within a block and making use of Cartier’s relation, isofactorial change of support support models are gained in the form of discrete point-block models [205, 207]. A case study on a lead-silver deposit is provided in an exercise (with solution).

The point-block-panel problem

The point-block-panel problem arose in mining, but it may easily be encountered in other fields. In mining exploration samples of a few cm$^3$ are taken at hectometric spacing and analyzed. Production areas are termed panels and are typically of hectometric size (say 100$\times$100$\times$5m$^3$). The panels are subdivided into basic production units, the blocks (say 10$\times$10$\times$5m$^3$). These blocks correspond in mining to the quantity of material an engine can carry and a decision will be taken whether the engine is directed to the plant for processing the material or else to the waste dump. During production several samples are taken in each block and only blocks with an average grade superior to a cut-off value $z_c$, above which they are profitable, will be kept.

The point-block-panel problem consists in anticipating before production, on the basis of the initial exploration data, what will be the proportion of profitable blocks within each panel, so a decision can be taken whether or not to start extraction of a given panel. Isofactorial change-of-support models and corresponding disjunctive kriging will make it possible to estimate different block selectivity statistics for individual panels.

The Figure 35.1 sketches a typical point-block-panel estimation problem. The three different supports involved are points $x$ denoting the exploration samples, blocks $v$ and panels $V$. In the model, the sample points $x$ will be considered as a randomly located in the blocks.

Cartier’s relation and point-block correlation

Suppose $x$ is a point located randomly inside a block $v$. Then the conditional expectation of the randomly located random function value knowing the block value is the block value,

$$E[Z(x) \mid Z(v)] = Z(v), \quad \text{(35.1)}$$

which is known as Cartier's relation.
To understand this relation we rewrite the conditional expectation making the two sources of randomness more explicit,

$$E[Z(x) \mid Z(V)] = E_x[ E_Z[Z(x) \mid Z(v)]]$$

(35.2)

$$= \frac{1}{|v|} \int_{x \in v} E[Z(x) \mid Z(v)] \, dx$$

(35.3)

Now the relation is easily demonstrated:

$$E[Z(x) \mid Z(v)] = E_Z \left[ \frac{1}{|v|} \int Z(x) \, dx \mid Z(v) \right]$$

(35.4)

$$= E[Z(v) \mid Z(v)] = Z(v).$$

(35.5)

For a point Gaussian anamorphosis $Z(x) = \varphi(Y(x))$ and a corresponding block anamorphosis $Z(v) = \varphi_v(Y(v))$ we have by Cartier’s relation,

$$E[\varphi(Y(x)) \mid Z(v)] = E[\varphi(Y(x)) \mid \varphi_v(Y(v))] = \varphi_v(Y(v)).$$

(35.6)