Under favorable circumstances the integral on the right side of Eq. (53.01) can be held at the order of \( LP^2/4 \). \( \text{Boehne and Jang} \) [1] calculate switching time (order of 0.01 to 0.02 sec) and total heat developed by the arc under several typical circumstances.

§ 54. Electric oscillations generated by \( d. c. \) arcs

Fig. (54.01) shows a circuit containing a capacitor \( C \), a resistance \( R \), and an inductance \( L \) in series with an arc. The source of the \( d. c. \) current through the arc is not designated. It could be the main circuit of Fig. (61.01) while the quench circuit would correspond to the circuit of Fig. (54.01). As to this circuit, it merely carries a current \( I \) that is an \( a. c. \) or a transient damped current; but, because of \( C \), the \( D. C. \) that maintains the arc does not enter this circuit. We consider a moment when the current in the circuit of Fig. (54.01) is \( I \), the capacity has the voltage \( V_c \) and the arc has the voltage \( V_a \). The electric state of the circuit is expressed by Eq. (54.02)

\[
V_c = L \frac{dI}{dt} + RI + V_a
\]

(54.02)

A short time \( dt \) later

\[
V_c + dV_c = L \frac{d(I + dI)}{dt} + R(I + dI) + V_a + dV_a
\]

holds. Subtraction of the first equation from the latter one gives

\[
dV_c = L \frac{d^2I}{dt^2} + RdI + dV_a
\]

We introduce \( Q \) the charge of \( C \)

\[
Q = V_c C \quad \text{and} \quad -\frac{dQ}{dt} = I
\]

Hence

\[
\frac{d^2I}{dt^2} + \frac{1}{L} \left( R + \frac{\partial V_a}{\partial I} \right) \frac{dI}{dt} + \frac{1}{LC} I = 0
\]

(54.03)

which has the form of the familiar equation of oscillations in a damped electric circuit. \( \frac{\partial V_a}{\partial I} \) is written as a partial differential quotient since \( V_a \) will depend also on the length \( s \) of the arc. The second term in
§ 54. Electric oscillations generated by d. c. arcs

(54.03) represents the damping. Writing

\[ r_1 = R + \frac{\partial V_\alpha}{\partial I} \]  

(54.04)

the condition for aperiodic variation becomes

\[ \frac{1}{LC} < \left( \frac{r_1}{2L} \right)^2 \quad \text{or} \quad \frac{L}{Cr_1^2} < \frac{1}{4} \]  

(54.05)

and the condition for oscillations

\[ \frac{1}{LC} > \left( \frac{r_1}{2L} \right)^2 \quad \text{or} \quad \frac{L}{Cr_1^2} > \frac{1}{4} \]  

(54.06)

In § 50, we have seen that stationary \( VI \)-characteristics of the arc have negative \( \partial V_\alpha/\partial I \). The influence of this fact on oscillations in a circuit as demonstrated in Fig. (54.01) may become clear by numerical examples. To begin with we refer to static characteristics and to Diagram XI, and afterwards discuss actual deviations from the conditions assumed.

Assumptions: \( R = 9 \ \Omega \), \( L = 10^{-5} \ \text{H} \), \( C = 2.5 \cdot 10^{-6} \ \text{F} \), constant length, \( s \), of the arc and the following three cases represented by points on arc characteristics

1. \( I - I_m = 1 \ \text{A} \), \( s = 0.03 \ \text{cm} \)
2. \( I - I_m = 0.64 \ \text{A} \), \( s = 0.04 \ \text{cm} \)
3. \( I - I_m = 0.4 \ \text{A} \), \( s = 0.05 \ \text{cm} \)

Diagram XI gives

<table>
<thead>
<tr>
<th>for case</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\partial V_\alpha}{\partial I} )</td>
<td>-3</td>
<td>-9</td>
<td>-23 \ \Omega</td>
</tr>
<tr>
<td>hence ( r_1 )</td>
<td>6</td>
<td>0</td>
<td>-14 \ \Omega</td>
</tr>
</tbody>
</table>

and according to Eqs. (54.05) and (54.06) oscillation undamped swinging up to larger oscillation oscillation and larger amplitude

In case (3) the amplitude of the oscillation would increase by the factor of 100 after one period – if the arc obeyed the static \( VI \)-characteristic. But it does not. In order to adapt itself to a change of voltage and current, the arc must change the heat content of its gas and vapor, adjust the cathode and anode spots, and so on; and the adjustment consumes time. Therefore, against variations of a very high frequency the arc tends to keep its resistance constant. If the arc actually succeeded in doing so the slope of its dynamic characteristic would become positive, \( \partial V/\partial I = V/I \). Against a frequency of \( 10^4 \ \text{Hz} \), the slope \( \partial V/\partial I \) may remain negative but will be smaller than that in Diagram XI.