

## CHAPTER 21

# Structural Changes in the Real GNP Interdependence of the USA, West Germany, and Japan: 1970–1986

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### Summary

This chapter first locates quarters in the early 1970s at which the cUovariance matrices of the innovation vectors have shifted for the real GNPs of the USA, West Germany, and Japan treated as univariate series. The chapter then exhibits differences in the impulse response time profiles of the two models estimated from the data primarily before and after the break as a concise summary of the changes in dynamic interactions of the three real GNPs.

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### 21.1 Introduction

This chapter uses the state space modeling method for “trending” time series, i.e., time series with roots close to one, to locate possible breaks in the real GNPs and money stock series of the USA, West Germany, and Japan by the likelihood ratio test. The real GNP series are first treated separately as univariate series to locate a likely quarter at which changes in some characteristics of the data-generating process have occurred.

The chapter then jointly treats the three real GNP series as a trivariate series. The focus in this part of the chapter is to examine the differences in the structure of dynamic interdependence by the time profiles of impulse responses (dynamic multipliers), rather than pinpointing the quarter of structural shifts.

Several methods have been proposed in literature to detect sudden (or gradual) changes in parameters of data-generating processes [Andrews and Fair (1988), Goldfeld and Quandt (1976), Lo and Newey (1985) in the econometric literature and Willsky and Jones (1976) and Basseville *et al.* (1987) in the systems literature]. We are interested in detecting structural changes in vector-valued macroeconomic time series, such as money stocks and real GNP. Most of the methods are for univariate series although some extension for vector-valued series are available.

In the context of state space innovation modeling of time series, the innovation vectors,  $e_t = y_t - y_{t|t-1}$ , where  $y_{t|t-1}$  is the orthogonal projection of  $y_t$  onto the manifold spanned by its own past data, are modeled as approximately normally distributed with mean zero and sample covariance matrix  $\Delta$ . The joint probability distribution of  $y_1, \dots, y_T$  has only  $\Delta$  as the parameters when the innovation representation is used.

A parameter shift in the data-generating process manifests itself then as changes in the covariance matrix  $\Delta$  of the innovation vector. [We need not adopt ad hoc assumptions on the breaks of "slope" or intercept points of the time series.] Given that a single shift in the covariance matrix has occurred in a sample period, we can adopt the method of Goldfeld and Quandt (1976) to locate the time instant which is the most likely instant of the parameter shift by maximizing the joint likelihood function over the sample period.

Suppose that  $t_c$  is the instant of the parameter shift so that  $\text{Cov}\{e_t\} = \Delta_1$  for  $t \leq t_c$  but  $\text{Cov}\{e_t\} = \Delta_2$  for  $t > t_c$ .

The joint likelihood function is

$$L(y_1, \dots, y_T | t_c) = \text{const } |\Delta_1|^{-t_c/2} |\Delta_2|^{(T-t_c)/2} \exp[-1/2 \text{tr}(\Delta_1 S_1 + \Delta_2 S_2)],$$

where

$$S_1 = \sum_{t=1}^{t_c} e_t e_t'$$

and

$$S_2 = \sum_{t=t_c+1}^T e_t e_t'.$$

The regime shift is identified with the  $t_c$ , which maximizes the joint likelihood function.

## 21.2 Univariate Series

Episodes in the late 1960s and early 1970s such as the demise of the Brettonwood accord and the oil shocks tell us that a regime shift is likely during a period spanning from the late 1960s to the early 1970s. The procedure outlined above is applied to the quarterly US money stock data from the first quarter 1947 [1947(I)] to 1982(II). The total of 141