8. Antiwaveguide Sound Propagation

In contrast to waveguide propagation, antiwaveguide propagation of sound occurs when a ray leaving a source never returns to the depth of its source. An example of antiwaveguide propagation is given in Fig. 1.8. Here we shall consider this kind of propagation for two different cases, i.e. depending on whether the velocity gradient $dc/dz$ at an antiwaveguide axis is nonzero (Sect. 8.1) or equals zero (Sects. 8.2, 3).

8.1 Linear Antiwaveguide Adjacent to Water Surface

Let us assume that in the half-space $z > 0$ bounded by a free water surface at $z = 0$, the square of the refractive index is given by the linear law

$$n^2(z) = 1 + az.$$  \hfill (8.1.1)

At small $az$ it approximately corresponds to the linear law for sound velocity $c(z)$ (Sect. 6.6). Equation (6.5.4) with $k(z) = k_0 n(z)$ is reduced to (6.6.12) if we set

$$t = t_0 - z/H,$$

where

$$t_0 = H^2 (\xi^2 - k_0^2), \quad H = (ak_0^2)^{-1/3}.$$  \hfill (8.1.2)

The sound field at an arbitrary point is again described by (6.6.6) with the properly chosen eigenfunctions $\psi_j(z)$.

For the antiwaveguide conditions these functions at $z \to \infty$ must represent outgoing waves. This condition is satisfied by the Airy function $Z(t)$ (Sect. 6.6) whose asymptotic representation as $z \to \infty$ is, by virtue of (6.6.14, 16),

$$Z(t) \sim (-t)^{-1/4} \exp \left[ i \left( v + \pi/4 \right) \right], \quad v = \frac{2}{3} \left( \frac{z}{H} - t_0 \right)^{3/2}.$$  \hfill (8.1.3)

The boundary condition requires that this function be zero at $z = 0$ (free water surface),
which gives the equation for eigenvalues $\xi_l$.

Taking into account the results obtained in Sect. 6.6, the solution of (8.1.4) may also be written as

$$t_0 = t_{0l} = y_l \exp (i \pi /3).$$

Using the relation between $t_0$ and $\xi$ (8.1.2), we obtain

$$\xi_l^2 = k_0^2 + (y_l/H^2) \exp (i \pi /3).$$

Thus, all $\xi_l$ are complex and, therefore, all waves are attenuated. Attenuation increases with the mode number $l$.

If we take into account that

$$\frac{\partial}{\partial z} = -\frac{1}{H} \frac{\partial}{\partial t}, \quad \frac{\partial}{\partial \xi} = 2 \xi H^2 \frac{\partial}{\partial t},$$

the acoustic pressure at point $(r, z)$ can be written using (6.6.6) as

$$p(r, z) = -\frac{i \pi}{H} \sum_l Z(t_l) Z(t_{1l}) \left[ \frac{\partial Z}{\partial t} \right]_{t_0l}^{-2} H_0^{(1)}(\xi_l r)$$

where

$$t_{0l} = y_l \exp (i \pi /3), \quad t_l = t_{0l} - z/H, \quad t_{1l} = t_{0l} - z_l/H.$$

As usual, the sound source is assumed to be at point $(0, z_1)$.

It is of interest to consider the asymptotic behaviour of (8.1.7) for $z_l, \xi_l \gg 1$. In this case $t_l$ and $t_{1l}$ are negative, and their moduli are large. Then, using asymptotic representations of the Hankel function and of the function $Z(t)$ (8.1.3), we have

$$Z(t_l) Z(t_{1l}) H_0^{(1)}(\xi_l r) = 2^{1/2} (\pi \xi_l r)^{-1/2} (t_l t_{1l})^{-1/4} \exp \left[i(v_l + v_{1l} + \xi_l r + \pi/4)\right],$$

where

$$v_l = (2/3)(z/H)^{3/2} (1 - t_{0l} H/z)^{3/2} \approx (2/3)(z/H)^{3/2} - (z/H)^{1/2} t_{0l},$$

$$v_{1l} \approx (2/3)(z_1/H)^{3/2} - (z_1/H)^{1/2} t_{0l},$$

$$\xi_l = (k_0^2 + t_{0l}/H^2)^{1/2} \approx k_0 + t_{0l} (2k_0 H^2)^{-1}.$$

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1 For reasons which will be clear in Sect. 8.2 these waves are called quasi-modes.