A GAME EQUILIBRIUM MODEL OF THIN MARKETS

by

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Abstract: We consider games of group, or coalition, formation occurring over infinite, discrete time, with new participants becoming active in the game in each period, and with participants that have successfully formed groups leaving the game each period. Markets may be "thin", in the sense that the number of participants active in the game in any time period is finite and may be small. We construct a subgame perfect equilibrium for an example and show some additional properties of the equilibrium. One property is that, even though markets are thin, the "first mover" within a time period has an advantage (and realizes more than a competitive payoff) only in special circumstances, and, along the equilibrium path, he is the only mover who can have such an advantage. Also, we discuss the limit behavior of the model as costs of waiting (time costs) become small; specifically, the equilibrium payoffs converge to core payoffs of a game with a continuum of players and finite coalitions (f-core payoffs). The static continuum game provides an idealization of the limit of the dynamic games for small waiting costs. Thus our research initiates providing a noncooperative foundation for the core as a solution concept for such games.

1. INTRODUCTION

Our motivation comes from "thin markets"-ones with possibly only a few participants in the market at any one time. It has been demonstrated that economies and games with many players and with the property that "small" groups of players can realize all gains to group formation are competitive, in the sense that cooperative outcomes coincide with price-taking equilibria. However, although "effective" groups may be small, such as buyer-seller pairs, these models require a large total player set. Our study is motivated by the observation that, at any point in time, there may be only a few participants in the market. Also, for many economic interactions, participants may not be confined to buy, sell, or make a deal immediately; they might be able to postpone reaching an agreement. For example, one might rent a temporary residence and postpone buying a house until next month, or next year, or longer. Even casual perusal of the "Homes for Sale" or "Business" sections of the newspapers indicates that the expected economic situation "next period" is a major determinant of current economic variables.

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(See Mas-Colell (1979), Kaneko-Wooders (1986, 1989), Hammond-Kaneko-Wooders (1989), and Wooders (1988)).
In this paper we investigate a highly stylized model of thin markets. The markets are described by a dynamic noncooperative model of group formation and payoff distribution. There is a countable number of time periods and, at most, a finite number of participants within each time period. A finite number of new participants become active in the game each period, and also, each period, there is the possibility of participants completing their market activities and leaving the game. Remaining in the market more than 1 period is costly. We show existence and provide a description of a subgame perfect equilibrium. One notable feature of the equilibrium is that only in special circumstances concerning the exact numbers of participants active in the market is there an advantage to being selected as the first mover, or decision-maker, within a time period. Moreover, we show that as time costs become small, the subgame perfect equilibrium payoffs converge to cooperative/competitive outcomes in the sense that the equilibrium payoffs are in the core of a related static game (a game with a continuum of players and finite coalitions.)

We develop one version of the model in detail, and provide a sketch of another. We sometimes refer to these as "examples", since we have in mind, and are currently working at a model which generalizes both versions in this paper. The first example in this paper has the advantage that it is sufficiently simple to enable us to construct an equilibrium point.

The games we study are ones of group formation and payoff distribution and are related to characteristic function bargaining games. For purposes of exposition, we will call any group that can realize a positive payoff a "firm" and use other terms consistent with this analogy, such as "workers", and "wages". However, it is not to be construed that the groups are necessarily "firms"; they could be "bridge clubs", for example, or "teams" or buyer-seller pairs. The activity of the game is described as occurring in a "market place". Since the total number of active participants in a period is finite, the games are "thin", in analogy with thin markets. The activity of the market-place is firm formation, and payoff distribution via wages and net revenues.

For our first example there are two permissible firm sizes, 1-member firms and n-member firms, where n is an exogenous parameter.$^3$ At the beginning of a time period, a new participant arrives at the market-place. There is then some finite number of participants in the market place throughout the current period. These participants are randomly ordered, and the $j^{th}$ participant in the ordering is called the $j^{th}$ decision-maker. The decision makers, each in accordance with his position in the ordering, successively choose alternatives from a set containing alternatives of four kinds. The kinds of alternatives are:

$^3$We will indicate the relaxation of this restriction.