A Theoretical Analysis of Global Illumination Models

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Abstract

This paper provides a general theoretical framework in which any previous global illumination model can be described. It gives a theoretical background which is based on radiometric quantities and on a physical approach to insure generality and consistency. A system of energy balance equations is obtained. Two approaches are possible depending on the way the radiometric properties of surfaces are characterized. Furthermore, the problem of solving this system of equations with the two-component method is addressed, and we show how we can take advantage of the characteristics of each component to select an appropriate method.

1 Introduction

In the recent literature, significant progresses have been made in the field of rendering techniques through the use of improved global illumination model. A global illumination model describes the light transport mechanism between surfaces, that is, how each surface element interacts with the others. Therefore, the global illumination model is a key problem when accuracy is needed in the rendering process (photorealism or photosimulation). So far, two approaches have been proposed to solve this problem: the former is based on ray tracing and the latter is based on the so-called radiosity solution. As these two approaches are complementary, several authors have tried to deal efficiently with all the light transport mechanisms by mixing both approaches in their model. Generally, these papers present their solution as an improvement to traditional ray tracing or radiosity approach and therefore they emphasize the algorithmical aspect to the detriment of the theoretical background. It is thus difficult to appreciate what assumption or what approximation the authors are making, which makes it difficult to compare one model with the other.

The primary goal of this paper is to provide a very general theoretical framework in which any global illumination model can be described. The rendering equation of Kajiya [10] is an attempt in this direction, however, the author did not use the usual radiometric quantities and some points remain unclear in his theory. In the following, all the theoretical background will be based on radiometric quantities to insure generality and consistency. Then, we will show how to make use of this theory to derive appropriate photometric algorithms.

This paper addresses the following subjects. First, the general system of equations describing the global energy balance is derived. Actually, two equivalent systems of equations can be stated depending on the used photometric quantity, namely light power and

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radiance. Starting from the systems of equations, the different classes of solutions are presented and discussed.

2 Global Energy Balance Equation

2.1 Generalities

Our goal is to derive a system of equations that describes how surfaces interact with each other. To be general, these equations must be based on the theory of radiometry and account for all type of light transport mechanisms [16]. The solution of this system provides the radiometric characteristics of all surfaces in the scene. However, these equations precisely depend on the radiometric quantity used to characterize the radiometric properties of surfaces.

Most well-known illumination models assume that the radiometric properties of surfaces are described by their radiances because the human eye is only sensitive to this quantity. The radiance (sometimes called intensity) is the light power leaving a surface per unit surface and per unit projected solid angle. The radiance value is a function of the coordinates of the point on the surface and of the direction of observation.

In the radiosity approach, the radiometric properties of surface are characterized in terms of power rather than radiance because radiosity takes its origin into the thermal radiation theory. Power is simply the light energy per unit time. However, it is still necessary to compute radiance at a final step to conform to the human eye perception. Radiance and power are related by the following relation (see figure 1):

\[
d^2 \Phi = L_1 \cos \Theta_1 dS_1 d\Omega_1 = L_1 \cos \Theta_1 \cos \Theta_2 \frac{dS_1 dS_2}{r^2}
\]

The application of this equation will be analysed in more details in the following, however it is already obvious that the evaluation of the radiance function from power implies the subdivision of all surfaces into small enough patches because of the differential nature of the above equation.

2.2 The Radiance Approach

2.2.1 The Light Transport Equation

Our goal is now to derive an expression of the light power leaving a surface \( S_i \) at a point \( P_i \) and reaching surface \( S_j \) at a point \( P_j \) (figure 2). This light power is due to the self emittance of \( S_i \) but also to its reflective properties. At this point, we leave out of account the occlusion problem and consider that any surface \( S_k \) contributes entirely to the illumination of surface \( S_i \).