A COMPARATIVE STUDY OF SEVERAL BOUNDARY ELEMENTS IN ELASTICITY

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INTRODUCTION

Although many elasticity problems have been solved by both analytical and numerical techniques, solutions for actual problems involving complex geometries are still relatively scarce.

For practical structural geometries, numerical methods are advantageous, but have some limitations in representing high stress gradients.

A general purpose Boundary Element program has been developed for elasticity problems, in which different elements can be easily implemented.

In this paper several boundary element solutions for elasticity problems are compared with finite element results and analytical solutions. The advantages and disadvantages of the boundary element method and of the different elements used are discussed with reference to the applications. Also, the principal features of the computer program are described.

ANALYTICAL BACKGROUND

For the sake of notation and completeness it is recalled that the boundary element method is based on the Somigliana's boundary identity, as presented by Brebbia and Walker (1980),

\[ c_{ij}(P)u_i(P) + \int_s \tau_{ij}(P,Q)T_{ij}(P,Q)ds = \int_s \sigma_{ij}(P,Q)u_i(P,Q)ds + \]

\[ + \int_V b_j(q)u_{ij}(P,Q)dV(q) \]  \hspace{1cm} (1)

where \( P, Q \) are points of the surface \( s \) of the domain \( V \). \( U_{ij} \) and \( T_{ij} \) are the fundamental Kelvin solutions, \( c_{ij} \) are coefficients depending on the geometry of the boundary at the point \( P \), and \( q \) is a point in the domain. Tensor notation is used in this expression and the indices have the range 1, 2, 3. When body forces...
are not considered equation (1) is only dependent on the surface displacements and tractions, $u_i$ and $t_i$ respectively.

The surface can be discretized into elements with a particular shape and a certain number of nodes, say $n$. Any field variable within each element is assumed to be given by

$$\theta(\xi) = M^\alpha(\xi)\theta^\alpha \quad \alpha = 1, 2, \ldots, n$$

or

$$\theta(\xi, \eta) = M^\alpha(\xi, \eta)\theta^\alpha \quad \alpha = 1, 2, \ldots, n$$

for 2D and 3D cases respectively, $M^\alpha$ are shape functions of local coordinates $\xi$ and $\eta$, $\theta^\alpha$ are the values of field variables at nodal points. If the surface displacements and tractions are introduced as field variables equation (1) becomes, for 2D problems

$$\frac{\partial}{\partial \xi}\left[ c_{\alpha\beta} u_{\alpha}(P_n) u_{\beta}(P_n) + \sum_{k=1}^{n} \int M^\alpha(\xi) T_{ij}(P_n, Q(\xi)) J(\xi) d\xi t^\beta_{ij} \right]$$

$$= \sum_{k=1}^{n} \int M^\alpha(\xi) u_{ij}(P_n, Q(\xi)) J(\xi) d\xi t^\beta_{ij}$$

where $n_2$ is the number of boundary elements

$u^\alpha_{ij}$ is the value of $u_{ij}$ at local node $\alpha$

$t^\beta_{ij}$ is the value of $t_{ij}$ at local node $\alpha$

$J$ is the Jacobian of transformation of coordinates.

Noting that $P_n$ refers to a particular node, then for all nodes equations (3) can be expressed in matrix form as follows:

$$[C]\{U\} + [H]\{U\} = [G]\{T\}$$

or

$$[H]\{U\} = [G]\{T\}$$

where $\{U\}$ and $\{T\}$ are the surface nodal displacements and tractions respectively. The elements of $[H]$ and $[G]$ can be obtained from the integrals

$$H^\alpha_{ij} = \int_{S_k} M^\alpha(\xi) T_{ij}(P_n, Q(\xi)) J(\xi) d\xi$$

$$G^\alpha_{ij} = \int_{S_k} M^\alpha(\xi) u_{ij}(P_n, Q(\xi)) J(\xi) d\xi$$

These integrals are normally integrated by standard Gaussian quadrature. When these integrands are singular they are calculated by rigid body considerations or by the technique described in the Appendix.

Applying the boundary conditions, equation (5) becomes

$$[A]\{x\} = \{f\}$$

where $\{x\}$ and $\{f\}$ are the vectors of unknown and known quantities respectively.

The displacements at interior points can be calculated using the following expression