11.1 Introduction

The basic laws of fluid dynamics are conservation laws; they are statements that express the conservation of mass, momentum and energy in a volume closed by a surface. Only with the supplementary requirement of sufficient regularity of the solution can these laws be converted into partial differential equations. Sufficient regularity cannot always be guaranteed. Shocks form the most typical flow situation with a discontinuous flow field. In case discontinuities occur, the solution of the partial differential equations is to be interpreted in a weak form, i.e. as a solution of the integral form of the equations. For example, the laws governing the flow through a shock, i.e. the Hugoniot–Rankine laws, are combinations of the conservation laws in integral form. It is clear that for a correct representation of shocks, also in a numerical method, these laws have to be respected.

There are additional situations where an accurate representation of the conservation laws is important in a numerical method. A second example is the slipline which occurs behind an airfoil or a blade if the entropy production is different on streamlines on both sides of the profile. In this case, a tangential discontinuity occurs. Another typical situation is an incompressible flow where the imposition of incompressibility, as a conservation law for mass, determines the pressure field.

In the cases cited above, it is extremely important that the conservation laws in their integral form are represented accurately. The most natural method to accomplish this obviously is to discretize the integral form of the equations and not the differential form. This is the basis of the finite volume method. It is also to be noted that, in cases where strong conservation in integral form is not absolutely necessary, it is still physically appealing to use the basic laws in their most primitive form.

In the purest form of the method, the flow field or domain is subdivided, as in the finite element method, into a set of non-overlapping cells that cover the whole domain on which the conservation laws are applied. (In the finite volume method (FVM) the term cell is used instead of the term element used in the finite element method (FEM).) On each cell the conservation laws are applied to determine the flow field variables in some discrete points of the cells, called nodes. As in the FEM, these nodes are at typical locations of the cells, such as cell-
Fig. 11.1. Typical choice of grids in the FVM, (a) structured quadrilateral grid; (b) structured triangular grid; (c) unstructured triangular grid.

Fig. 11.2. Typical choice of nodes in the FVM. The marked nodes indicate the nodes intervening in the flux balance of the control volume. (a) piecewise constant interpolation structure; (b) piecewise linear interpolation structure; (c) no interpolation structure with all variables defined in each node; (d) no interpolation structure with not all variables defined in each node (Cartesian grid). ● : \( \rho \) and \( p \), □ : \( u \), □ : \( v \).

centres, cell-vertices or mid-sides. Obviously, there is considerable freedom in the choice of the cells and the nodes. Cells can be triangular, quadrilateral, etc. They can be elements of a structured grid or an unstructured grid. Clearly, the whole geometrical freedom of the FEM can be used in the FVM. Figure 11.1 shows some typical grids.

The choice of the nodes can be governed by the wish to represent the solution by an interpolation structure, as in the FEM. A typical choice is then cell-centres for representation as piecewise constant functions or cell-vertices for representation as piecewise linear (or bilinear) functions. However, in the FVM, a function space for the solution need not necessarily be defined and nodes can be chosen in a way that does not imply an interpolation structure. Figure 11.2 shows some typical examples of choices of nodes with the associated definition of variables.