ACQUISITION OF PROOF SKILLS
IN GEOMETRY

John R. Anderson
Carnegie-Mellon University

ABSTRACT

The ACT theory of learning is applied to the domain of high school geometry. The concern is with how students become skilled at planning a proof of a geometry problem. A general control structure is proposed for integrating backward and forward search in proof planning. This is embodied in a production system framework. Two types of learning are described. Knowledge compilation is concerned with how students transit from a declarative characterization of the domain to a set of operators for performing a specific task in that domain. Tuning is concerned with how students learn which problem features are predictive of the success of which operators.

7.1 INTRODUCTION

Much of my research has been concerned with the refinement of skills from general methods. I have developed a theory of learning called ACT which involves a set of mechanisms by which skills can be refined. These include knowledge compilation mechanisms for converting from declarative representation of a skill to a procedural representation. They also include a set of mechanisms for learning which problem features are predictive of the success of problem-solving operators. Much of the later discussion in this chapter is concerned with describing these mechanisms and their application to acquisition of proof skills in geometry. First, however, I will discuss the nature of the empirical phenomena that we are trying to simulate with our learning system and how our performance theory goes about organizing search for geometry proofs.

We have been studying how high school students learn to generate proofs
in geometry and how they get better at generating proofs through practice. A major empirical base for this work comes from protocols of thirty 45-minute sessions that we had with one of our students (Subject R). In these sessions the student read textbook instructions and worked out textbook problems. We tried to confine our interruptions to clearing up serious misconceptions. R did all of his work in these sessions; his textbook and notes were taken away from him, and he was encouraged not to think about geometry between sessions. Thus, we have a more or less complete record of the learning that occurs in the first part of geometry. In the thirty sessions he worked through two column proofs, a section about angles, to where he was generating non-trivial proofs about triangle congruence. A substantial amount of learning occurs after this initial period. Therefore, we have supplemented our data base with spot protocols from more advanced high school students and from various adults who are relatively expert at generating geometry proofs.

Our goal has been to generate a computer simulation of the learning processes in geometry. The ultimate test of this program is that it be given textbook instruction and have a learning history like that of our high school student. The dimensions of this ultimate test are, of course, a little overwhelming. For the time being we have contented ourselves with simulating learning on fragments of the geometry text. A major concern in this research has been the so-called "sufficiency condition" for a psychological theory—that is proposing mechanisms powerful enough to produce the observed learning of the necessary skills.

The constraint that the behavior of the system be such that it corresponds to human behavior is a severe one but not one that is orthogonal to the frequent AI goal of getting a system capable of intelligent behavior. We have argued elsewhere [Anderson & Kline, 1977] that the psychological constraint may facilitate ultimately achieving a robust intelligent system, particularly if the goal is machine learning. Therefore, I would commend to the reader the learning proposal contained in here as a viable scheme for skill acquisition by a machine.

The simulation has been worked out in the context of the ACT system [Anderson, 1976] which is a simulation system based on hypotheses about the basic mechanisms of human cognition. The procedural knowledge of the ACT system is based on a production system architecture and the declarative component in ACT is based on a semantic network. The productions use the information in the semantic network as a working memory to match against. The learning investigations discussed in this chapter are principally focused on how new productions are developed in acquiring a skill. The ACT theory has been tested out on a wide variety of empirical domains including memory and inferential processes [Anderson, 1976], language acquisition and processing [Anderson et al., 1977], [Anderson, 1981], and schema abstraction and prototype formation [Anderson et al., 1979].