Maximum Likelihood Estimation for the VAR-VARCH Model: A New Approach

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Abstract

We consider a general multivariate conditional heteroskedastic time series model and derive the information matrix of the maximum likelihood estimator by using the matrix differential calculus techniques of Magnus and Neudecker (1991). We discuss the VAR-VARCH model as a special case, and demonstrate the maximum likelihood estimation of the information matrix in an example with simulated data.

Key words: VAR-VARCH model; volatile time series; maximum likelihood; information matrix; matrix differential calculus.

Introduction

Since Engle (1982) and Bollerslev (1986) the literature on ARCH (autoregressive conditional heteroskedastic) and GARCH (generalized ARCH) models has been growing rapidly. Applications in economics and finance for volatile time series can be found in, e.g. Bollerslev, Chou and Kroner (1992), Mills (1993), Bera and Higgins (1995) and Palm (1996). We refer to Engle and Kroner (1995) for alternative multivariate generalized ARCH models, Geweke (1989) for Bayesian studies on ARCH and generalized ARCH models, Polasek and Kozumi (1996) for a Bayesian approach treating a VAR-VARCH model (vector autoregressive model with ARCH errors) in the random coefficient framework.

Wong and Li (1997) generalizes the so-called CHARMA (conditional heteroskedastic autoregressive moving-average) model in the univariate case of Tsay (1987). For the multivariate CHARMA model, Wong and Li (1997) uses the so-called star product, see, e.g. MacRae (1974) and Roger (1980),
to derive the information matrix for maximum likelihood estimation. The star product is introduced and used for the chain rule in matrix differential calculus. But its application to deriving and presenting results can be complicated in some situations. We notice that when using the standard concept and techniques of matrix differential in, e.g. Magnus (1988) or Magnus and Neudecker (1991) it is advantageous to implement the chain rule in good notations and to make calculations shorter and more efficient. See also e.g. Liu (1995) for recent results and applications in econometrics. In the present paper, we will use the standard techniques to study a general multivariate time series model, which is parameterized by only conditional mean and conditional variance and which contains the CHARMA model and the VAR-VARCH model as special cases. As we will see, the analysis of the model is not only shorter and more general, but it will also reveal that the information matrix of the CHARMA model in Wong and Li (1997) misses some terms and therefore is not correct.

The plan of this paper is as follows: In Section 2, we outline the normal log-likelihood function and the maximum likelihood estimation procedure and present the information matrix. In Section 3, we analyze the VAR-VARCH model in a special case and give the associated version of the information matrix. In Section 4, we discuss a numerical example with simulated data. In Section 5, we summarize the paper with some final remarks. In the appendix, the derivation of the information matrix can be found.

**Maximum likelihood estimation for heteroskedastic time series**

Consider a general multivariate conditional heteroskedastic time series model of the following form

\[ y_t = \mu_t + u_t, \quad t = 1, \ldots, T, \]  

where \( u_t \) is an \( M \times 1 \) disturbance vector, \( y_t \) is a vector of time series of dimension \( M \), i.e., a realization of an independently and identically normally distributed stochastic process which is characterized by both the \( M \times 1 \) conditional mean vector \( E(y_t \mid \psi_{t-1}) = \mu_t \) and \( M \times M \) conditional variance matrix \( Var(y_t \mid \psi_{t-1}) = H_t \), where \( \psi_{t-1} \) indicates the realized values of the conditional information set, \( \mu_t \) is an \( M \times 1 \) vector and \( H_t \) is an \( M \times M \) positive definite matrix.