Testing for a Long Run Relationship between Trend and Difference Stationary Series

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The theory of cointegration and error correction models developed in the last couple of years by, among others, Engle and Granger (1987), as well as Johansen (1988) proved very useful for testing and estimating long-run relationships between difference stationary [I(1)] series. However, there is an increasing literature expressing the view that the autocorrelation of macroeconomic time series could be produced either by trend or difference stationary models [e.g. Christiano Eichenbaum (1990) and the evidence presented by Stock (1991)]. Of course, the idea of cointegration cannot be applied to trend stationary series, but there is the related idea of codependence suggested by Gourieroux and Jourdain (1989) which is designed for testing a long run relationship between stationary series. This paper uses this approach in a VAR framework which can be applied to trend and difference stationary variables.

Consider the following VAR model for the vector of series $X_t$, including intercept and deterministic trend terms:

$$X_t = \mu + b t + A_1 X_{t-1} + \ldots + A_k X_{t-k} + \epsilon_t$$

We have the impulse response function

$$X_t = \delta + \gamma t + \sum_{\tau=0}^{\infty} \theta_{\tau} \epsilon_{t-\tau}.$$ 

Now consider an equilibrium relationship $\beta' X_t$, which adjusts more quickly to shocks than the original series. This means that $\beta' \theta_{c+1} = 0$ (i.e. $i = 0, 1, \ldots$) holds for some $c > 0$ and $\theta_{c+1} \neq 0$ (i.e. $i = 0, 1, \ldots k$). In addition, we assume that $\beta' X_t$ has no deterministic
trend.

If the series are stationary, these restrictions hold for a relatively low value of \( c \) in order to make them codependent. For cointegrated \( I(1) \) series, they are approximately valid for a relatively large value of \( c \), which depends on the adjustment speed.

These restrictions can be tested most conveniently by considering the regression

\[
\beta'X_t = \beta'd + \beta'ct + \beta'\Gamma X_{t-c} + \ldots + \beta'\Gamma X_{t-c-k+1} + \nu_t
\]

which is obtained by repeated substitution of lagged \( X_t \) in the VAR equation and taking \( \beta'X_t \). The error term \( \nu_t \) is, therefore \( \beta'(c_t + \theta_1c_{t-1} + \ldots + \theta_{c-1}c_{t-c+1}) \). If \( \beta'X_t \) fully adjusts to shocks after \( c \) periods and if it has no deterministic trend, all \( p \cdot k + 1 \) coefficients (except the intercept) of this equation should be zero. This hypothesis can be conveniently tested by an OLS based Wald statistic correcting the covariance matrix for MA \( (c-1) \) error by an approach proposed by Levine (1983). According to the work of Park and Phillips (1989), the test should be asymptotically valid for \( I(1) \) series too, as the RHV's are cointegrated. In practical application, the test has to be repeatedly applied for reasonable values of \( c \) (see Kugler and Neusser, 1992). In addition, it should be mentioned that this test is easily extended to more than one equilibrium relationship.

The Monte Carlo experiments reported by Kugler and Schwendener (1991) as well as Kugler and Neusser (1992) indicate that the test works reasonably well for mean stationary series. The conjecture that it can also be applied to trend stationary series and cointegrated series is confirmed by the Monte Carlo results obtained in the framework of this paper.