Regular Path Expressions in Feature Logic*

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Abstract. We examine the existential fragment of a feature logic, which is extended by regular path expressions. A regular path expression is a subterm relation, where the allowed paths for the subterms are restricted by a regular language. We will prove that satisfiability is decidable. This is achieved by setting up a quasi-terminating rewrite system.

1 Introduction

Feature descriptions are used as the main data structure of so-called unification grammars, which have become the predominant paradigm for natural language processing (for a good introduction see [Shi86]). More recently, feature descriptions have been proposed as a constraint system for logic programming (e.g. see [ST92]). They provide for a typical partial description of abstract objects by means of functional attributes called features. As an example consider the feature description (in matrix notation)

\[
x : \exists y \begin{bmatrix}
    \text{woman} \\
    \text{father} : \begin{bmatrix}
        \text{engineer} \\
        \text{age} : y
    \end{bmatrix}
\end{bmatrix},
\begin{bmatrix}
    \text{husband} : \begin{bmatrix}
        \text{painter} \\
        \text{age} : y
    \end{bmatrix}
\end{bmatrix},
\]

which may be read as saying that \( x \) is a woman whose father is an engineer, whose husband is a painter and whose father and husband are both of the same age.

Feature descriptions have been proposed in various forms with various formalizations. We will follow the logical approach introduced by Smolka [Smo88, Smo92], where feature descriptions are standard first order formulae interpreted in first order structures. In this formalization features are considered as functional relations. Atomic formula (which we will call atomic constraints) are of the

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form $A(x)$ or $xfy$, where $x, y$ are first order variables, $A$ is some sort predicate and $f$ is a feature (written infix notation). Using this notation, we can express the above feature description by the (less suggestive) formula

$$\exists y, x_1, x_2 \ ( \text{woman}(x) \land x \text{father } x_1 \land \text{engineer}(x_1) \land x_1 \text{ age } y) \land x \text{husband } x_2 \land \text{painter}(x_2) \land x_2 \text{ age } y).$$

This feature logic has been investigated in detail. A complete axiomatization of the standard model (the so-called feature graphs) is given in [BS92]. There it was shown, that the standard model is elementarily equivalent to a tree model. Additionally, some connection to first order constructor terms has been examined [ST92].

In this paper we will be concerned with an extension to feature descriptions, which has been introduced under the notion of “functional uncertainty” by Kaplan and Maxwell [KM88]. This extension is done by adding a subterm relation, where the allowed paths for the subterms are required to be in a given regular language. It was invented for handling so-called long-distance dependencies in the grammar formalism LFG [KB82]. For a detailed description the reader is referred to [KZ88]. Further applications can be found in [Kel91].

For this extension we first have to generalize the constraints of the form $xfy$ to constraints of the form $xwy$, where $w = f_1 \ldots f_n$ is a string of features (called feature path). The feature paths are interpreted using simple relational composition.

As Smolka [Smo88] shows, this generalization is just syntactic sugar. This changes if we add functional uncertainty in form of constraints $xLy$, where $L$ is a regular language of feature paths. A constraint $xLy$ holds if there is a path $w \in L$ such that $xwy$ holds. By this existential interpretation a constraint $xLy$ can be seen as the disjunction

$$xLy = \bigvee \{xwy \mid w \in L \}.$$

Because this disjunction can be infinite, functional uncertainty yields additional expressivity. Note that the constraint $xwy$ can also be expressed by $x\{w\}y$.

Kaplan and Maxwell [KM88] have shown that the satisfiability problem of the pure existential fragment (i.e. the satisfiability of formulae built with $A(x)$, $xLy$ and equations $x \doteq y$) is decidable, provided that a certain acyclicity condition is met. Baader et al. [BBN+91] have shown, that satisfiability is undecidable if negation is added. But it is an open problem whether satisfiability of the pure existential fragment without any additional conditions (such as acyclicity) is decidable. In this work we show that this is indeed decidable, thus filling this gap.

2 The Method

At first we will briefly describe the main part of solving standard feature descriptions and then turn over to the extension by functional uncertainty. To get