Improving Transformation Systems for General $E$-Unification

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Abstract. In this paper we motivate and present a new and improved transformation system for general $E$-unification. It can be seen as a modification of the original transformation system by Gallier and Snyder refined by ordinary unification and basic paramodulation. We present a short proof of completeness. Besides completeness we can also show an important property of the transformation system which is not known for the original system: independence of the selection rule. This motivates the abstraction of transformation sequences to equational proof trees thus obtaining static proof objects which facilitates finding further refinements of the procedure.

1 Introduction

Transformation systems are a rather recent approach to general $E$-unification (e.g. [MMR86], [GS89], [Sny91]). Unlike completion based approaches to $E$-unification (e.g. [HR87], [BDP89]), which (for the general case) have to proceed in a bottom-up fashion, transformation systems operate in a goal-oriented manner. They solve $E$-unification problems by repeated transformation of ‘complex’ problems to ‘less complex’ ones. The price to be payed for the goal-orientedness first of all is that it is not possible to impose ordering constraints on the application of equations. Secondly, in order to be complete without the need to paramodulate into variables, the application of equations must be done lazily. This means, that instead of immediately unifying the subterm to be rewritten and the term on one side of an equation, a new $E$-unification problem to be solved is generated. Unfortunately, this leads to a system of transformation rules with a considerably large search space.

Despite the fact that in the meantime some effort has been made to improve transformation systems (e.g. [DJ90b]), up to now they were not attractive enough to be applied for example in automated theorem proving. In addition to the huge search space involved, a reason could be that it has not yet been proven, whether transformation systems are independent of the selection rule. Although this is commonly assumed, in order to guarantee completeness during the search for solutions one would have to examine all possible orderings of selections of goals, since transformation systems were just shown to be nondeterministically complete.
In this paper we want to attack both the inherent inefficiency of transformation systems and the unsolved question of independence of the selection rule. We motivate and present a refined transformation system which is characterized by ordinary unification and the restriction of applications of equations to basic terms. The basic restriction (for details cf. e.g. [Hul80], [NRS89], [BGLS92]) serves to abstract whether a variable is instantiated or not, and ordinary unification replaces three rules for unification in the original transformation system by Gallier and Snyder. We can show that we are still complete and have a substantially reduced search space. Moreover, we can prove the important property that the completeness of the proposed transformation system is independent of the selection rule. We not only show that for different selection rules (variants of) the same E-unifiers can be obtained, but moreover, that we can get the same structure of the derivation of an E-unifier. This allows us to abstract transformation sequences to proof objects: basic equational proof trees. A basic equational proof tree will be defined as a representative for a class of transformation sequences (disregarding their selection rules) and it represents the structure of the actual proof. This abstract view facilitates the recognition and proof of properties and refinements of the transformation system more easily than in the original sequence setting. We will demonstrate its significance on a first proposition about equivalent transformations which leads to a further refinement of the system.

Our intention with this paper is twofold. First, we want to present an improved general and complete transformation system which satisfies certain important properties for its application in automated theorem proving (like the independence of the selection rule). The second point is to show that the established properties are the basis for a variety of straightforward proofs of improvements. The transformation system and its refinements in this paper are a first step in this direction.

2 Preliminaries

This section provides a short sketch of some of the basic concepts and definitions used in this paper. A more detailed description can be found e.g. in [DJ90a] or [JK91]. The notational conventions we use were adopted from [DJ89] and [BGLS92].

An equation is a pair of terms related by the special (symmetric) predicate symbol \( \simeq \). A closure \( e \cdot \sigma \) is a pair consisting of an skeleton \( e \) and a substitution \( \sigma \). The skeleton can be an arbitrary structure, e.g. an equation or a multiset of equations. Closures will serve to express the notion that no equation may be applied to a term or subterm introduced by a substitution (known as basic restriction, e.g. [Hul80]; for more details on closures cf. [BGLS92]).

A position \( p \) in a term \( t \) is represented by a sequence of positive natural numbers. The set of all positions in a term \( t \) is denoted by \( \text{Pos}(t) \), the set of non-variable positions by \( \mathcal{FP}(t) \). The top-most position in a term is \( \Delta. \ t[p \) represents the subterm of \( t \) at the position \( p \) and \( t[s]_p \) the result of replacing the subterm in \( t \) at position \( p \) by the term \( s \). By \( p \parallel q \) we denote that the positions \( p \)