Algorithms for Bicriteria Combinatorial Optimization Problems

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1 Introduction

In recent years, many types of interactive optimization methods have been developed in order to support multicriteria decision makings (see the book by Sawaragi, Nakayama and Tanino, 1985 and Wierzbicki and Lewandowski, 1987). Given a ***feasible decision set*** $X \subseteq R^n$, and p objective functions, $f_1, f_2, \ldots, f_p$ (all are assumed to be minimization for convenience), the following problem has been used in various situations of interactive multicriteria decision makings:

$$\underset{x \in X}{\text{minimize}} \max_{1 \leq i \leq p} \{\alpha_i f_i(x) + \beta_i\}, \tag{1}$$

where α_i and β_i are positive and real constants respectively, which are computed based on the information supplied by the decision maker and/or the decision support system. For example, α_i and β_i are determined from aspiration and reservation levels in the reference point method which is one of the well known methods used in interactive multicriteria decision support systems (see Wierzbicki and Lewandowski, 1987, for the survey of reference point methods).

In view of this, it is of great significance to study the computational complexity required for solving the problem (1).

We concentrate on the case where $p = 2$ and each single objective problem P_i, $i = 1, 2$ defined below

$$P_i: \quad \underset{x \in X}{\text{minimize}}\ f_i(x) \tag{2}$$

is a minimum cost circulation problem (SMCP). Both problems are assumed to have optimal solutions. We shall study problem (1) with such restrictions, which we call a ***bicriteria minimum-cost circulation problem*** (BMCP). Given a directed graph $G = (V, E)$, where V and E denote the sets of vertices and edges respectively, a single objective

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minimum-cost circulation problem (SMCP) can be written as follows.

$$\text{SMCP}: \quad \text{minimize} \sum_{e\in E} c(e)x(e) \tag{3}$$

subject to

$$\left\{\sum x(e) \mid e = (u,v) \in E\right\} = \left\{\sum x(e') \mid e' = (v,w) \in E\right\} \text{ for } v \in V \tag{4}$$

$$a(e) \le x(e) \le b(e) \text{ for } e \in E. \tag{5}$$

Here $a(e)$, $b(e)$ and $c(e)$ are given integer numbers. $a(e) = -\infty$ and $b(e) = +\infty$ are allowed. Hence the feasible set X of SMCP and BMCP is the set of vectors $\{x(e) \mid e \in E\}$ satisfying (4) and (5).

Let the objective functions f_1 and f_2 for Problem BMCP be

$$f_1(x) = \sum_{e\in E} c_1(e)x(e) \quad \text{and} \quad f_2(x) = \sum_{e\in E} c_2(e)x(e), \tag{6}$$

and define

$$g_i(x) = \alpha_i f_i(x) + \beta_i, \qquad i = 1,2, \tag{7}$$

where $c_1(e)$ and $c_2(e)$ are integers.

Recently Tardos (1985) discovered a strongly polynomial algorithm for SMCP. Roughly speaking, an algorithm is called *strongly polynomial* if the running time is polynomially bounded only in the number of input data but not in the input size (see Tardos, 1986, for the precise definition of "strongly polynomial"). The best known strongly polynomial algorithm for SMCP is due to Galil and Tardos (1986), which runs in $0(n^2(m + n\log n)\log n)$ time, where $n = |V|$ and $m = |E|$. See (Fujishige, 1986) and (Orlin, 1984) for other versions of strongly polynomial algorithms for SMCP.

The major goal of this paper is to propose a strongly polynomial algorithm for solving Problem BMCP.

Notice that BMCP can be equivalently transformed to the following form:

$$\begin{aligned} \text{BMCP'}: \quad & \text{minimize } z \\ & \text{subject to } x \in X \text{ and} \\ & \qquad g_i(x) \le z, \quad i = 1,2\ . \end{aligned}$$

Such formulation has been used in the more general setting in order to solve problem (1) (see Chapter 7 of the book by Sawaragi, Nakayama and Tanino, 1985). This approach may not be recommended in case the set X has a good structure, since the new constraints $g_i(x) \le z$, $i = 1,2$ added to the original feasible decision set X may destroy the good structure of X. In our problem, we cannot guarantee any more the total unimodularity of the constraint matrix associated with the constraints for the above problem BMCP'.

The algorithm proposed here, on the other hand, does not use the above formulation, but takes full advantage of the good structure of the constraints (4) and (5). It employs as a subroutine the strongly polynomial algorithm for solving Problem SMCP by Galil and Tardos (1986) and finds an optimal solution of Problem BMCP in