resistance $R$ when the voltage $V_a$ is applied to it, which corresponds to the design of the dashed curve. For higher voltages its resistance is smaller than $R$ and it is evident that the dashed curve touches a characteristic for a shorter $s$ than the touching point with the straight resistance line $R$ determines. In other words, replacing $R$ by a semiconductor (without changing the resistance for $V_a$) shortens the life of the arc.

**I. Vacuum arc.** We speak of a vacuum arc if vacuum existed around the electrodes when the arc was ignited (for instance, drawn) and if no gas be admitted during the lifetime of the arc. Of course, the arc burns in the vapor issued from the electrodes. The rate of evaporation is essentially the same as with an arc burning in air; see Fig.(64.03), where the material transferred per coulomb is plotted against current intensity for arcs in air and arcs in vacuum. The plots arrange themselves along the same curve. But the absence of gas different from the vapor issued from the electrodes has the following consequences. On the one hand, when the current is interrupted the vapor rapidly condenses on the electrodes, and the gap quickly assumes a high dielectric strength, see § 50C. On the other hand, in the vacuum arc the plasma region has a low pressure, at least in case of a moderate current, and consequently the voltage gradient is small in the plasma. This fact is evidenced by the $VI$-characteristics for various widths, $s$, of the gap approaching each other as is demonstrated by Fig.(53.05).

Evidently the supply of vapor will be richer the lower the melting and vaporization points of the metal. In conformity with the theory Reece [1] found that a cadmium cathode allowed a 2 amp arc to burn steadily whereas an arc with a wolfram cathode was unstable even at 50 amp.

**§ 54. Electric oscillations generated by d–c arcs**

Fig.(54.01) shows a circuit containing a capacitor $C$, a resistance $R$, and an inductance $L$ in series with an arc. The source of the $d$–$c$ current through the arc is not designed. It could be the main circuit of Fig.(60.01) while the quench circuit would correspond to the circuit of Fig.(54.01). As to this circuit, it merely carries a current $I$ that is an $a$–$c$ or a transient damped current; but, because of $C$, the $d$–$c$ that maintains the arc does not enter this circuit. We consider a moment when the current in the circuit of Fig.(54.01) is $I$, the capacity has the voltage $V_c$ and the arc has the voltage $V_a$. The electric state of the circuit is expressed by Eq.(54.02)

$$V_c = L \frac{dI}{dt} + RI + V_a$$

(54.02)

A short time $dt$ later

$$V_c + dV_c = L \frac{d(I + dI)}{dt} + R(I + dI) + V_a + dV_a$$
holds. Subtraction of the first equation from the latter one gives

\[ \frac{d V_c}{dt} = L \frac{d^2 I}{dt^2} + R \frac{d I}{dt} + \frac{1}{LC} I = 0 \]

where we introduce \( Q \) the charge of \( C \)

\[ Q = V_c C \quad \text{and} \quad \frac{d Q}{dt} = I \]

Hence

\[ \frac{d^2 I}{dt^2} + \frac{1}{L} \left( R + \frac{\partial V_a}{\partial I} \right) \frac{d I}{dt} + \frac{1}{LC} I = 0 \]  \hspace{1cm} (54.03)

which has the form of the familiar equation of oscillations in a damped electric circuit. \( \frac{\partial V_a}{\partial I} \) is written as a partial differential quotient, since \( V_a \) will depend also on the length \( s \) of the arc. The second term in (54.03) represents the damping. Writing

\[ r_1 = R + \frac{\partial V_a}{\partial I} \]  \hspace{1cm} (54.04)

and considering the length \( s \) of the arc to be constant during the time considered the condition for aperiodic variation is

\[ \frac{1}{LC} < \left( \frac{r_1}{2L} \right)^2 \quad \text{or} \quad \frac{L}{Cr_1^2} < \frac{1}{4} \]  \hspace{1cm} (54.05)

and the condition for oscillations

\[ \frac{1}{LC} > \left( \frac{r_1}{2L} \right)^2 \quad \text{or} \quad \frac{L}{Cr_1^2} > \frac{1}{4} \]  \hspace{1cm} (54.06)

In §53, we have seen that stationary \( VI \)-characteristics of the arc have negative \( \frac{\partial V_a}{\partial I} \). The influence of this fact on oscillations in a circuit as demonstrated in Fig. (54.01) may become clear by numerical examples. To begin with we refer to static characteristics and to Diagram XI, and afterwards discuss actual deviations from the conditions assumed.

Assumptions: \( R = 9 \Omega; \ L = 10^{-5} \text{ Hy}, \ C = 2.5 \cdot 10^{-6} \text{ F}, \) and the following three cases represented by points on arc characteristics

(1) \( I - I_m = 1 \text{ A}, \quad s = 0.03 \text{ cm} \)
(2) \( I - I_m = 0.64 \text{ A}, \quad s = 0.04 \text{ cm} \)
(3) \( I - I_m = 0.4 \text{ A}, \quad s = 0.05 \text{ cm} \)

Diagr. XI gives

<table>
<thead>
<tr>
<th>for case</th>
<th>(1) ( \frac{\partial V_a}{\partial I} )</th>
<th>(2) ( \frac{\partial V_a}{\partial I} )</th>
<th>(3) ( \frac{\partial V_a}{\partial I} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\partial V_a}{\partial I} )</td>
<td>-3</td>
<td>-9</td>
<td>-23 \Omega</td>
</tr>
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</table>

hence

\[ r_1 \]

and according to

<table>
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<tr>
<th>Eqs. (54.05) and (54.06)</th>
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<tbody>
<tr>
<td>aperiodic undamped swinging up to larger oscillation oscillation and larger amplitude</td>
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</table>

\[ ^1 \text{We use} \ r_1 \text{ with the index 1 in order to obtain conformity to the designations in} \ § \ 60. \]