Wave propagation, dispersion and energy transport
in arterial blood flow

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1. Introduction

This paper presents a description of arterial blood flow from the viewpoint of a fluid dynamicist.

In this paper it is suggested that one may consider the peripheral flow as being driven by the radiation pressure of the pulse wave. This is only a terminological innovation, since the process is qualitatively well known by workers in medicine, but has been vaguely stated and largely ignored by fluid dynamicists concerned with physiological problems.

Radiation pressure is used here just as a catchword for the effect of progressive waves upon the mean flow momentum. It implies that one has to consider finite amplitude effects.

In the presence of fluctuations, the mean pressure drop may decrease, and it may even change sign.

It is rather likely that this effect is important in the circulatory system. The pulse wave transmits energy to the peripheral parts of the arterial tree. As the wave decays, its energy is transmitted to the mean flow, the radiation pressure difference times the mean velocity represents this energy transfer in the region of wave decay.

Additionally, the waves may lower the mean friction in the larger arteries, and thus the mean pressure drop.

It is of course possible also to introduce a deterministic argument here, reminding oneself that the circulatory system is a product of an evolutionary process, and therefore represents some kind of a biologically possible optimum, therefore the fluctuations must serve a function in propelling the blood through the system.

2. The finite amplitude pulse wave

One may think of the pulse wave as consisting of a sudden jump (a “shock” wave), followed by an expansion wave, on which there are sine-waves travelling.

The mechanism of propagation of a finite step in arterial dilatation does not involve any interesting acoustic effects, the speed of sound in blood and tissue being nearly equal.

The analysis consists of picking a coordinate system in which the phenomenon is steady, and requiring that mass, momentum and energy shall be preserved.

Fig. 1 shows the variables involved. The upper picture shows the variables in a coordinate system where the artery wall has no longitudinal velocity, the lower part of the figure shows the coordinates in which the waves are fixed.

In wave coordinates, the velocity of the artery wall is $-W$, the blood velocities before and after the wave $V_1$ and $V_2$, respectively, and the corresponding tube radii $R_1$ and $R_2$. Ignore viscous effects, and assume one-dimensional flow.
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The continuity equation is:

\[ \pi R_1^2 V_1 = \pi R_2^2 V_2. \]  

Bernoulli's equation reads:

\[ V_1^2 - V_2^2 = \frac{2}{\rho} (P_2 - P_1). \]

Energy conservation requires that the work done by the pressure to dilate the artery is supplied by the fluid:

\[ W \int_{R_1}^{R_2} P dA = W \int_{R_1}^{R_2} 2\pi R P dR = \frac{\rho}{2} \pi R_1^2 V_1 (V_2^2 - V_1^2). \]  

A relation between pressure \( P \) and radius \( R \) is still needed. For a thin walled elastic tube with radius \( R_0 \) at zero pressure difference across the wall and wall thickness \( h \), one finds:

\[ P = \frac{E h}{R_0} \left( 1 - \frac{R_2}{R} \right), \]

where \( E \) is the modulus of elasticity of the wall material. One then obtains:

\[ W \frac{\pi E h}{R_0} \left( (R_2 - R_0)^2 - (R_1 - R_0)^2 \right) = \frac{\rho}{2} R_1^2 V_1 (V_2^2 - V_1^2). \]

Combining Eqs. (2.1), (2.2), (2.4) and (2.6) one finds:

\[ V_1^2 = \frac{E h}{2\rho R_1} \left( \frac{R_2^2 + R_1^2}{R_2^4 + R_1^4} \right) \left( \frac{4 R_2 R_1}{R_2^4 + R_1^4} \right)^4 \]

\[ W = \frac{R_1}{R_2 (R_2 + R_1 - 2 R_0)} V_1 = \frac{R_1}{R_2 + R_1 - 2 R_0} \sqrt{\frac{4 R_2 R_1}{R_2^4 + R_1^4} \left( \frac{R_2^2 + R_1^2}{R_2^4 + R_1^4} \right)} \frac{E h}{2\rho R_0}. \]

For waves of infinitesimal amplitude, the energy equation does not apply, being of second order, and Eq. (2.7) gives:

\[ V_1^2 = \frac{E h}{2\rho R_1} = c_1^2 (\text{say}), \]

which is the Moens-Korteweg formula for water hammer. \( V_1 \) is the propagation speed of the wave with respect to the fluid (Moens 1876).

The permissible wave amplitude. For the case of the finite pulse wave, the velocity ahead of the wave \( U_1 = W - V_1 \) is given, and \( R_1 \) and \( R_2 \) are specified by the given pressures \( P_2 \) and \( P_1 \).

Eqs. (2.7) and (2.8) are then not independent, but relate \( R_2 \) and \( R_1 \).

Self-preserving waves are thus possible only for certain amplitudes.

Dilatational wave progressing into fluid at rest. For zero fluid velocity \( U_1 \) ahead of the wave, one finds

\[ U_1 = V_1 + W = 0 \]

(from Fig. 1). Eq. (2.8) then gives

\[ V_1 + W = \left[ 1 - \frac{R_2 R_1}{R_2 (R_2 + R_1 - 2 R_0)} \right] V_1 \]

or

\[ \frac{R_2}{R_0} = 1 - \frac{R_1}{2 R_0} \pm \sqrt{1 + \left( \frac{R_1}{2 R_0} \right)^2} \]

1 Eq. (2.3) implies that the power loss to surrounding tissue, branchings and other processes not accounted for in the analysis is: \((P_2 - P_1) V_1 A_1\), an assumption which is reasonable in form, but may be wrong in magnitude.