3 Solution of Nonlinear Equations

3.1 Static Equilibrium Position

In this chapter we are seeking an equilibrium position of a multibody system. This is a position where the system does not move. The velocity and acceleration are zero. There are mainly two different ways to transfer a given multibody system into an equilibrium state:

- to adjust the nominal forces in such a way that a given position becomes an equilibrium position;
- to modify the position for given nominal forces, so that with the modified position the system is in an equilibrium state.

We saw in Sec. 1.6 that the first task normally requires the solution of a system of linear equations. The second task yields a system of nonlinear equations of the form

\[ F(x) := \begin{pmatrix} f_a(p, 0) - G^T(p)\lambda \\ g(p) \end{pmatrix} = 0 \quad (3.1.1) \]

with \( x^T = (p^T, \lambda^T) \).

The determination of such an equilibrium position is important

- because it contains interesting information about the system’s mechanical behavior;
- in order to obtain a point for linearization to perform a linear system analysis;
- in order to obtain consistent initial values for the integration of the equations of motion of the system.

Solving nonlinear systems is not only interesting in this context, but it is also a subtask of many other problems, e.g. simulation, optimization.
Example 3.1.1 If for the unconstrained truck example the individual bodies are assembled without respecting the influence of gravitational forces $m_i g_{gr}$, i.e. we assume $g_{gr} = 0$, the configuration shown in Fig. 3.1 is obtained. This configuration is called an assembling configuration, as it is the configuration which is described in a CAD environment by just assembling the individual construction elements without considering masses and forces. Adding gravitational forces moves the system into its equilibrium position represented by the dotted lines in Fig. 3.1.

Figure 3.1: Assembling position vs. equilibrium position of the unconstrained truck

Often, there is a free parameter $s$ in the description of the problem and (3.1.1) takes the form

$$F(x, s) = \left( f_a(p, 0, s) - G^T(p, s) \lambda \right) - g(p, s) = 0.$$  \hfill (3.1.2)

Then, one is interested either in the curve $x(s)$ or in the solution for a particular parameter value $s_1$. In the example above this value is $s_1 = g_{gr}$.

Example 3.1.2 We construct a parameter dependent solution for the unconstrained truck when increasing $s$ from 0 to $g_{gr} = 9.81 \text{ m/s}^2$. This is shown in Figs. 3.2.

Another typical example for a one-parametric problem is the determination of an equilibrium position for a mechanical system with a reference system which moves with constant speed along a curve with constant curvature. The curvature introduces centrifugal forces into the system, which affect the equilibrium position.