Chapter 1.
Plastic Deformation of Metals.

1.1. Phenomenological Theories.

The classical theory of plasticity is based on the concept of a yield criterion \( f(\sigma_{ij}) = k \), where \( \sigma_{ij} \) is the stress tensor and \( k \) is a material constant, which is in general a function of the previous strain history. For an isotropic material, the function \( f \) may be expressed in terms of the three invariants \( I_1, I_2, I_3 \), of the stress tensor. Experimentally, it is found that the hydrostatic component of stress does not affect plastic flow, so that the stress may be replaced by the stress deviation

\[
S_{ij} = \sigma_{ij} - \frac{1}{3} \sigma \delta_{ij},
\]

where \( \sigma = \sigma_{kk} \) and \( \delta_{ij} \) is the Kronecker delta. The yield criterion thus reduces to \( f(J_2, J_3) = k \), where \( J_2, J_3 \) are the second and third invariants of \( S_{ij} \) (\( J_1 \) being identically zero). In the von Mises criterion this is further simplified to \( J_2^{1/2} = k \) where \( k \) is now identified with the yield stress in simple shear.

For a material which work-hardens isotropically, \( k \) may be expressed as a function of the work of plastic deformation

\[
W = \int \sigma_{ij} \, d\varepsilon_p^{ij},
\]
where $\varepsilon_{ij}^p$ is the plastic component of strain. For a non-work-hardening or ideally plastic material, $k$ is a constant.

The plastic strain-rate $\dot{\varepsilon}_{ij}^p$ is governed by the flow rule, according to which

$$\dot{\varepsilon}_{ij}^p = \lambda \frac{\partial f}{\partial \sigma_{ij}} ,$$

where $\lambda$ is a scalar factor of proportionality. This rule determines only the relative plastic strain rates; the stress-strain relation is independent of time.

The total strain rate is given by adding the elastic component to the plastic component:

$$\dot{\varepsilon}_{ij} = \frac{1}{2G} \dot{\sigma}_{ij} + \frac{1 - 2\nu}{E} \sigma_m \delta_{ij} + \lambda \frac{\partial f}{\partial \sigma_{ij}} ,$$

where $\sigma_m = \sigma_{kk}/3$, $G$ is the shear modulus, $E$ is Young's modulus, and $\nu$ is Poisson's ratio.

To generalize (1.1.4) for rate-dependent materials, the concept of a visco-plastic material may be introduced. Such a material behaves elastically for stresses such that $f(\sigma_{ij}) < k$ but requires that $f(\sigma_{ij}) > k$ for finite rates of plastic strain. Following Sokolovsky [1] and Malvern [2], it may be assumed that the plastic strain rate is a function of the overstress, i.e. the amount by which the applied stress exceeds that corresponding to the flow stress for the same strain at vanishingly small strain rates.

Perzyna [3] has proposed the following constitu-