Threshold Behaviour of Ionization Cross-Sections

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3.1 Introduction

A relationship that is well-known in atomic physics is the Wannier law

\[ \sigma_{\text{ion}} \propto E^{1.127}. \]  (3-1)

It gives the dependence of the cross-section for the electron-impact ionization process

\[ e + A \rightarrow A^+ + e + e \]  (3-2)

or the photo-double-detachment process

\[ h\nu + A^- \rightarrow A^+ + e + e \]  (3-3)

on the amount \( E \) by which the energy of the system exceeds the ionization energy of the atom or negative ion. It is a threshold law, since it applies only when \( E \) is small, and it was first derived by Wannier (1953) using a treatment based on classical mechanics. It has excited, and continues to excite, considerable interest and some controversy, and has been the subject of many theoretical and experimental studies. A recent experimental result (which will be discussed in Section 3.2.4) is shown in Fig. 3-1.

Part of the attraction of the Wannier law is that it concerns a problem that lies at the interface of classical mechanics and quantum mechanics, and so is related to a wider range of problems for which there is, at present, no generally accepted method of solution. These problems are characterised by the fact that they involve interactions between three or more particles over distances that are too large for conventional quantum-mechanical techniques to be tractable. They usually also involve motion in the vicinity of an unstable potential ridge and the existence of an exceptionally high degree of correlation between the particles involved (see for example Fano 1980a, b, 1983a, b, Rau 1982). The subject of near-threshold ionization is particularly important since it represents the simplest example of a problem that contains all these features. The hope is therefore that the search for a theoretical technique
Fig. 3-1. Cross-section for the photodetachment process $h\nu + H^- \rightarrow H^+ + e + e$. The errors are statistical only. $a$ The curve is the best fit to the power law $\sigma \propto (E + \delta)^n$, where $E$ is the excess energy above the threshold at 14.352 eV (and where $\delta$ and $n$ are found to be $47 \pm 5$ meV and $1.15 \pm 0.04$ respectively). $b$ The curve is the best fit to Temkin's modulated linear law (equation (3-101)). From Donahue et al. (1982, 1984)

that will give a good description of the threshold ionization process will also lead to a better understanding of the wider class of related problems. It may also lead to a better understanding of non-threshold ionization, since although the long-range correlations emphasized by the Wannier theory produce their most conspicuous effects within a few eV of the ionization threshold they may also play a significant role at higher energies. Neglect of these correlations may therefore be responsible for the marked weakness of almost all theoretical techniques in the range up to approximately 50 eV above threshold.

Yet another part of the attraction of the law and of the associated aspects of near-threshold ionization and excitation processes is of course that the experimental tests are particularly difficult and exacting, usually requiring good energy resolution, high sensitivity and the ability to handle electrons of very low energy.

The aim of this chapter is to discuss the various derivations and extensions of the Wannier law, and to compare the theoretical results with the available experimental evidence.