Chapter VIII

Waves and Shock Waves in Radiation Gasdynamics

1. Introduction. In order to consider the influence of thermal radiation on the flow field, we study the wave motion of infinitesimal amplitude in a radiating gas. Such wave motion will bring out many essential features of radiation gasdynamics. In section 2, we discuss the wave motion of infinitesimal amplitude in an optically thick medium at very high temperature so that the gas is in a state of plasma. We consider the wave motion with the influence of both thermal radiation and electromagnetic forces. In section 3, we consider the wave motion in a radiating gas of finite mean free path of radiation. This case has not been thoroughly investigated as the case of optically thick medium. However some essential features will be discussed for this general case.

Since the radiation effects are most important in extremely high speed flow in which shock waves usually occur, the shock wave phenomena are the most important phenomena in radiation gasdynamics. In the next four sections, we deal with the shock waves in radiation gasdynamics. First the well known Rankine-Hugoniot relations will be extended in radiation gasdynamics. Then the effect on the shock wave structure by thermal radiation will be studied. Finally we consider the flow behind shock waves in radiation gasdynamics.

2. Wave of small amplitude in an optically thick medium. We consider a gas which is in a very high temperature state so that it is ionized and radiating. We assume that the gas is of sufficient opacity that the radiation can be considered as being trapped in it and as in equilibrium with the plasma. Hence the fundamental equations of radiation gasdynamics of small mean free path of radiation discussed in chapter V, § 8 hold true for our case. Since the gas is ionized, electromagnetic forces should be considered. These electromagnetic forces are determined by electromagnetic fields which are governed by the Maxwell’s equations of electromagnetic fields. In addition to the radiation gasdynamic variables: $q$, $p$, $\rho$, and $T$, we have also the electromagnetic variables: electrical field strength $\vec{E}$, magnetic field strength $\vec{H}$, electric current density $\vec{J}$ and electrical excess charge $\rho_e$. These electromagnetic variables are governed by the following equations (9).

Maxwell equations for the electromagnetic fields are

\[ \nabla \times \vec{H} = \vec{J} + \frac{\partial \rho_e \vec{E}}{\partial t} \quad (8.1) \]

\[ \nabla \times \vec{E} = -\frac{\partial \mu_e \vec{H}}{\partial t} \quad (8.2) \]

where $\mu_e$ is the magnetic permeability and $\epsilon$ is the inductive capacity.

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The generalized Ohm’s law is
\[ \mathbf{J} = \sigma_e (\mathbf{E} + \mathbf{q} \times \mathbf{B}) + \rho_e \mathbf{q} \] (8.3)
where \( \sigma_e \) is the electric conductivity of the gas and \( \mathbf{B} = \mu_e \mathbf{H} \) is the magnetic induction.

The conservation of the electrical charge gives
\[ \frac{\partial \rho_e}{\partial t} + \nabla \cdot \mathbf{J} = 0 \] (8.4)

The effects of electromagnetic variables on the equations of radiation gasdynamics (5.23) are:

(i) The electromagnetic force \( \mathbf{F}_e \) in equation (5.23c), i.e.,
\[ \mathbf{F}_e = \rho_e \mathbf{E} + \mathbf{J} \times \mathbf{B} \] (8.5)

and (ii) The electromagnetic energy flux \( Q \) in equation (5.23d), i.e.,
\[ Q = \mathbf{J} \cdot \mathbf{E} \] (8.6)

Now we assume that originally the plasma is at rest with a pressure \( p_0 \), temperature \( T_0 \) and density \( \rho_0 \) and that it is subjected to an externally applied uniform magnetic field \( \mathbf{H}_0 = i H_x + j H_y + k H_z \) where \( i, j, \) and \( k \) are respectively the x-, y-, and z-wise unit vectors; and \( H_x \) and \( H_y \) are constants. There is no electric current, nor excess electric charge, nor externally applied electric field. The plasma is perturbed by a small disturbance so that in the resultant disturbed motion we have:
\[ u = u (x, t), \quad v = v (x, t), \quad w = w (x, t), \]
\[ p = p_0 + p' (x, t), \quad T = T_0 + T' (x, t), \]
\[ \rho = \rho_0 + \rho' (x, t), \quad \mathbf{E} = \mathbf{E} (x, t), \]
\[ \mathbf{H} = \mathbf{H}_0 + \mathbf{h} (x, t), \quad \mathbf{J} = \mathbf{J} (x, t), \quad \rho_e = \rho_e (x, t) \] (8.7)

where \( u, v \) and \( w \) are respectively the perturbed x-, y- and z-velocity components; \( p', \rho' \) and \( T' \) are respectively the perturbed pressure, density and temperature; and \( \mathbf{E}, \mathbf{h}, \mathbf{J} \) and \( \rho_e \) are respectively the perturbed electric field strength, magnetic field strength, electric current density and the excess electric charge. We assume that all the perturbed quantities are small so that the second or higher order terms in these quantities are negligible. For simplicity, we assume that the perturbed quantities are functions of one space coordinate \( x \) and time \( t \) only. Thus we discuss only wave propagation in the direction of x-axis.

Substituting equation (8.7) into the fundamental equations of radiation magnetogasdynamics, i.e., equations (5.23) and (8.1) to (8.6) and neglecting the higher order terms of perturbed quantities, we have the following linear equations for the wave motion in radiation magnetogasdynamics: