Visualization of Generalized Voronoi Diagrams

Alexandru Telea, Jarke J. van Wijk

Eindhoven University of Technology,
Den Dolech 2, Eindhoven 5600 MB, The Netherlands,
{alext, vanwijk}@win.tue.nl

Abstract. Voronoi diagrams are an important data structure in computer science. However well studied mathematically, understanding such diagrams for different metrics, orders, and site shapes is a complex task. We propose a new method to visualize k-order diagrams and give an efficient adaptive implementation for this method. The algorithm is easy to customize for different metrics and site shapes. Its real-time performance makes it suitable for interactive planning and analysis of complex Voronoi configurations in 2D. We illustrate the method for different combinations of metrics and site shapes.

1 Introduction

Voronoi diagrams are a fundamental data structure in computer science. Mostly used in computational geometry, Voronoi diagrams have found their way in many application areas, such as computer graphics (collision detection, motion planning), optimization theory (associative file searching, clustering, scheduling), and physics (crystal and cell growth studies).

Voronoi diagrams based on Euclidean distance are the best known. Such diagrams partition a 2D plane in regions such that all points within a region are closest to one site from a given site set. Visualizing such diagrams is straightforwardly done by drawing the set of disjoint, adjacent planar polygons that represent the diagram.

We present a new visualization method for two generalizations of the Voronoi diagrams. The first generalization regards k-order diagrams, which partition the plane in cells such that all points in a given cell have the same k closest sites. Although many studies cover the mathematics of k-order Voronoi diagrams [2, 1], getting an intuitive understanding of such diagrams is a difficult task. Specifically, we would like to answer questions such as 'which are the k sites that influence a given partition' and 'which are all points under the k-order influence of a given site' in a simple, visual manner. The second generalization concerns using different metrics besides the Euclidean distance and different site shapes besides points. Diagrams for higher orders, different site shapes and metrics lead to complex shape-site relationships that require a more elaborate visualization than a straightforward polygonal drawing.

In Section 2, we give a mathematical overview of Voronoi diagrams and outline the difficulties inherent to their visualization. In Section 3, we introduce our method for visualizing generalized Voronoi diagrams and illustrate it with several examples. Section 4 presents an efficient implementation of the method. We conclude in Section 5 with future research directions.
2 Background

We begin with a description of the elementary properties of 2D Voronoi diagrams. More on the mathematical aspects is available from several surveys [2, 4, 5].

Let \( P = p_1, ..., p_n \) be a set of \( n \) distinct points in the plane, called sites, and \( d(p, q) \) the Euclidean distance between points \( p \) and \( q \). The first order Voronoi diagram of \( P \) is a subdivision of the plane in \( n \) cells, one for each site in \( P \), such that a point \( q \) lies in the cell of a site \( p_i \) if and only if \( d(q, p_i) < d(q, p_j) \) for all sites \( p_j \in P \) with \( j \neq i \). The cell boundaries lie thus on the perpendicular bisectors of the line segments \( p_i p_j \) (Fig. 1 a).

First order Voronoi diagrams are used, for example, to partition a city map into regions (cells), given a set of fire station positions (sites), such that any city location is assigned to the closest fire station.

![Fig. 1. Order-1 (a) and order-2 Voronoi diagram of a point set](image)

A k-order Voronoi diagram subdivides the plane in cells such that all points in a cell have the same ordered set of \( k \) closest sites from the site set \( P \). In our example, a k-order diagram would indicate which are the second, third, and \( k \)-th fire stations that serve a given city location if the closest (first order) fire station is unavailable for some reason. Conversely, one can visualize the regions served by a given fire station when some neighboring stations fail to work. In an interactive city planning setup, one could add, delete, or move the locations of the fire stations to optimize the area coverage and redundancy in case of fire station failure.

However, visualizing k-order diagrams by simply drawing the involved cells produces hardly readable drawings (Fig. 1 b). It is hard to tell from such a drawing which is the area served by a fire station if other stations fail or, conversely, which are the \( k \) stations serving a given city location. These questions could be answered by interactively selecting a site and highlighting its influence region or, conversely, by highlighting all stations that serve a given city location. However, such a method is not capable of producing an overview of the influence of all fire stations on all city locations simultaneously.

A second generalization of the Voronoi diagrams involves the distance function used. The \( L_1 \) (Manhattan distance) metric \( d(p, q) = |x_p - x_q| + |y_p - y_q| \) is used to model access times to strategic locations in a city where the streets form an orthogonal grid, or the access time to given records in mass storage systems where the read/write head can only move in orthogonal directions [3]. All edges in such a Voronoi diagram are vertical,