Subdivision Surfaces for Scattered-data Approximation

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Abstract. We propose a modified Loop subdivision surface scheme for the approximation of scattered data in the plane. Starting with a triangulated set of scattered data with associated function values, our scheme applies linear, stationary subdivision rules resulting in a hierarchy of triangulations that converge rapidly to a smooth limit surface. The novelty of our scheme is that it applies subdivision only to the ordinates of control points, whereas the triangulated mesh in the plane is fixed. Our subdivision scheme defines locally supported, bivariate basis functions and provides multiple levels of approximation with triangles. We use our subdivision scheme for terrain modeling.

1 Introduction

Subdivision surfaces [4, 8] are widely used for modeling surfaces of arbitrary topological genus. They are defined by polygonal control meshes that are recursively refined analogously to knot insertion for B-spline surfaces [15]. This refinement process converges to smooth limit surfaces that are in many cases piecewise polynomials. The subdivision schemes by Catmull/Clark [7] and Doo/Sabin [9], for example, reproduce uniform B-Splines on regular, rectilinear meshes.

The strength of subdivision surfaces is their ability to deal with irregular meshes defining arbitrary two-manifolds. Extraordinary points, i.e., surface points corresponding to vertices with other than four adjacent edges in a control mesh, are typically surrounded by an infinite number of smaller and smaller polynomial patches satisfying certain continuity constraints. Eigen-analysis of local subdivision matrices can be used to compute surface normals and to evaluate a limit surface at arbitrary parameter values [25,23,14].

Multiresolution modeling techniques, like wavelet transforms [26], are required for real-time visualization of large-scale data sets. Subdivision surfaces and wavelet transforms can be combined to a single, highly efficient multiresolution modeling tool [20,24,18,3]. Subdivision-surface wavelets with finite filters have been constructed for compression and multiresolution representation of functions defined on planar tessellations and surfaces of arbitrary topology like isosurfaces [1–3]. Related multiresolution methods [12,17] have been designed for completely irregular triangulated surfaces.
without subdivision connectivity. Wavelets and subdivision techniques are also successfully being applied to computational fluid dynamics (CFD) and flow visualization problems [27,21,6].

Despite of their simplicity and flexibility for modeling surfaces of arbitrary topology, subdivision surfaces have never been used for an apparently simpler problem—representing graph surfaces, i.e., modeling smooth functions defined on planar domains. Instead, piecewise polynomial constructions like the Clough-Tocher and Powell-Sabin interpolants [15] are frequently used, splitting every triangle into multiple macro-triangles that may have bad aspect-ratios. Additionally, interpolation constraints may cause unwanted variations that are not present in surfaces defined by control points without interpolation, like B-Splines. Other approaches are based on multiquadrics [10,11]. Some aspects of subdivision surfaces, however, have been exploited by a scattered-data fitting method based on triangular B-splines [22].

Classical visualization problems, like terrain modeling, have not taken advantage of subdivision techniques, in the past. In this paper we propose a simple and efficient variant of Loop’s subdivision scheme [19,26] for modeling scattered data in the plane. We expect that our subdivision technique will successfully be used for applications like terrain modeling and that trivariate constructions can be developed for volume modeling data defined on tetrahedral grids, as well.

Fig. 1. Loop subdivision process. Starting with a triangulated control mesh (octahedron, left), a hierarchy of triangulations converges to a smooth limit surface (right).

2 Loop’s Subdivision

We now review Loop’s subdivision scheme [19] generalizing quartic box splines [15] to arbitrary triangular control meshes. The big deal about subdivision schemes is that they generate smooth surfaces from irregular control meshes with extraordinary points. Extraordinary points correspond to vertices that have other than four incident edges in a locally rectilinear mesh and vertices that have other than six incident edges in a triangle mesh. Thus, it is possible to define smooth surfaces of arbitrary topology by simple control meshes.