Model Study of Bremsstrahlung in Alpha Decay

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Abstract. Recent experiments have triggered an interest in bremsstrahlung associated with \(\alpha\) decay. We propose a fully time-dependent quantum mechanical calculation, treating the electromagnetic interaction by first-order perturbation theory, of the bremsstrahlung probability coincident with \(\alpha\) decay. Preliminary results indicate that model calculations of this type are feasible.

1 Introduction

During the past decade two experiments measuring coincident bremsstrahlung and \(\alpha\) decay have been reported [1]. The interpretation of the measurements were discussed subsequently [2] and further experiments are indicated. The measurements were the impetus of a number of theoretical studies. The simple classical model of an \(\alpha\) particle accelerating in the Coulomb field overestimates the bremsstrahlung yields by an order of magnitude [3]. It suggests a non-negligible contribution to the radiation due to the \(\alpha\) particle tunneling through the barrier. In a quasi-classical analysis [4] the photon emission amplitude is obtained by treating the tunneling as a classical motion in complex time.

In a fully quantum mechanical calculation, Papenbrock and Bertsch [5] used Fermi's golden rule regarding the electromagnetic interaction. They assumed appropriate wave functions of the \(\alpha\) particle in the initial and final states but the time-dependent aspect of the \(\alpha\) decay process was not taken fully into account. They found only a small contribution to the bremsstrahlung due to tunneling. A more extensive study along these lines [6] suggests that the bremsstrahlung spectrum is a result of interference of a number of effects, such as, tunneling, Coulomb acceleration, as well as others. An alternative approach [7] involves the time-dependent wave function of an emitted \(\alpha\) particle which tunnels through a repulsive barrier to calculate the expectation value of the radial momentum.

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\ †Supported by the Natural Sciences and Engineering Research Council of Canada.
of the $\alpha$ particle as a function of time. This expectation value, in an ad hoc
manner, is used in the equation for classical bremsstrahlung. The resulting
bremsstrahlung spectrum is suppressed. It shows peaks due to the interference
of different resonance components of the $\alpha$ wave function.

The method we propose in this paper deals in full with the time-dependent
quantum aspects of the $\alpha$ decay and the accompanying radiation process. The
electromagnetic interaction, however, is treated by first-order perturbation the-
yory. As far as we know no such calculation has been done to date.

2 Description of the Model

The Hamiltonian for an $\alpha$ decaying system and the radiation field is $H =
H_0 + H_1$ with

$$H_0 = \frac{p^2}{2M} + V(r) + \sum_{\lambda=1,2} \int d^3 k \omega a_\lambda^\dagger(k) a_\lambda(k),$$  \hspace{1cm} (1)

where $M$ is the mass, $p = -i\nabla$ the momentum, and $r$ the coordinate of the
$\alpha$ particle. The potential $V(r)$ includes a repulsive barrier and supports one
or more unstable bound states of the $\alpha$ particle within the barrier. The $a_\lambda^\dagger(k)$
creates a photon of momentum $k$, energy $\omega = k = |k|$, and polarization $\epsilon_\lambda$. We
use natural units, $c = \hbar = 1$, and the radiation gauge for the photon field. The
interaction is given by $H_1 = -(2e/M)p \cdot A(r, t)$, where

$$A(r, t) = \sum_\lambda \int d^3 k f(k) \epsilon_\lambda [a_\lambda^\dagger(k)e^{i(k \cdot r - \omega t)} + a_\lambda(k)e^{-i(k \cdot r - \omega t)}].$$ \hspace{1cm} (2)

The $2e$ is the charge of the $\alpha$ particle and $f(k) = [(2\pi)^3(2\omega)]^{-\frac{1}{2}}$.

The state vector of the entire system can be written as $\Psi = \Psi_0 + \Psi_1$ with
$\Psi_0 = \psi_0(r, t)|0\rangle$ and

$$\Psi_1 = \frac{2e}{M} \sum_\lambda \int d^3 k f(k) \psi_1(k, \lambda, r, t)a_\lambda^\dagger(k)e^{i(k \cdot r - \omega t)}|0\rangle,$$

where $|0\rangle$ is the photon vacuum. The Schrödinger equation $(i\partial/\partial t - H)\Psi = 0$
can be reduced to

$$\begin{bmatrix}
i \frac{\partial}{\partial t} - \frac{p^2}{2M} - V(r) \end{bmatrix} \psi_0(r, t) = 0,$$

where we consider only an $S$ state for the $\alpha$ wave function $\psi_0(r, t)$, and

$$\begin{bmatrix}
i \frac{\partial}{\partial t} - \frac{p^2}{2M} - V(r) - \omega \end{bmatrix} \psi_1(k, \lambda, r, t)e^{i(k \cdot r - \omega t)} = -(p \cdot \epsilon_\lambda)\psi_0(r, t)e^{i(k \cdot r - \omega t)}.$$ \hspace{1cm} (5)

We ignore terms of order $e^2$. In Eq. (5), $\partial/\partial t$ and $p$ operate on $e^{i(k \cdot r - \omega t)}$ also.
The $\psi_1(k, \lambda, r, t)$ depends on angles through $(k \cdot r)$ and $(\epsilon_\lambda \cdot r)/r$. 