This talk describes results of a paper with the same title done jointly with H. Epstein and J. Fröhlich. Similar work has been done simultaneously and independently by J. Dimock, K. Osterwalder, and R. Sénéor, and since they also report at this conference I will not describe any of their methods.
I. Discussion of General Axiomatic Properties

The point of view we want to take is to derive properties of the scattering matrix in relativistic quantum field theory from properties of Schwinger functions (Euclidean Green's functions). To fix the notations, let me first introduce the objects we shall talk about. We consider most of the time, for convenience of exposition, a Wightman theory (with isolated mass $m$) depending only on the time variables, called nevertheless $x_1$.

Let $\omega^T(z)$ denote the analytic, truncated Wightman function, analytic in $U_\pi T_\pi$, $\pi$ the permutations of $\{1,..n\}$,

$$\mathcal{G}_\pi = \{z = x+iy ; \ y_1 < \ y_2 < \ldots < \ y_n \} \text{, (the permuted tube)}$$

with boundary value $\omega^T_\pi(x)$ in $\mathcal{G}_\pi$. At Euclidean points we have a function

$$S^T(y_1,\ldots,y_n) = \omega(iy_1,\ldots,iy_n), \quad (1)$$

the truncated Schwinger function (at non coinciding points!)

Given $\pi$, let $\xi_j^\pi = z_j^\pi - z_{(j+1)}^\pi = \xi_j^\pi + i\eta_j^\pi$, $j = 1,\ldots,n-1$.

Suppose that time ordered functions

$$\tilde{h}_\pi(x_1,\ldots,x_n) = \sum_{j=1}^{n-1} \Theta(\xi_j^\pi) \omega^T_\pi(x_1,\ldots,x_n)$$

can be defined and let $\delta(\Sigma p_j)h_\pi(p)$ be their Fourier transforms. Set $h(p) = \tilde{h}_\pi(p)$. Suppose furthermore the theory has a positive mass $m$. Then the S-matrix elements (if they exist) are given by

$$\langle p_1^{in} \ldots p_k^{in} p_{k+1}^{out} \ldots p_n^{out} \rangle$$