On the $\mathbb{Z}_2$ Lattice Higgs System

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Abstract

Some aspects of the phase structure of the $\mathbb{Z}_2$-lattice Higgs system are studied by means of convergent expansions.
1. Introduction

In these notes we study one of the Wegner's generalized Ising systems [1], a simple model for a gauge theory on a lattice according to Wilson's ideas [2]. Mainly, we discuss some results of joint works with G. Gallavotti and F. Guerra [3] and with R. Marra [4] concerning this system. More general lattice gauge theories are treated in other lectures at this conference and we refer to [5,6] for a general outline and results related to the present study.

The system can be described as follows. Let there be given a cubical lattice \( L \). To each bond \((i,j)\) of neighboring sites a variable \( A_{ij} = A_{ji} \) taking values in \( \mathbb{Z}_2 = \{-1, +1\} \) is assigned. A configuration \( A \) of the system is specified by giving the values of \( A_{ij} \) at each bond. An elementary square of four adjacent bonds is a plaquette \( p = (i,j,k,l) \) and we write \( A(p) = A_{ij}A_{jk}A_{kl}A_{li} \).

Let \( B \) be a given configuration on \( L \setminus A \) where \( A \) is finite. The energy (or the euclidean action in the field theory language) of the system restricted to \( A \) with boundary condition \( B \) outside \( A \) is

\[
H^B_A(A) = -\left( \sum_{\{i,j\} \cap A \neq \emptyset} A_{ij} + \beta_p \sum_{p \cap A \neq \emptyset} A(p) \right)
\]

(1)

where the configuration \( A \) coincides with \( B \) outside \( A \). The coefficients \( \beta_L \) and \( \beta_p \) are the coupling constants.

A gauge invariant version of this system can be presented by introducing extra \( \mathbb{Z}_2 \)-valued variables \( K_i \) at each site and replacing \( \beta_L \sum A_{ij} \) by \( \beta_L \sum K_i A_{ij} K_j \). In this form the system can be viewed as a gauge field theory on the lattice, the Higgs and the Yang-Mills fields being respectively described by the variables \( K_i \) and \( A_{ij} \). However, the gauge transformation \( A_{ij} \rightarrow K_i A_{ij} K_j \) eliminates the \( K \)'s while leaving the plaquette term invariant, and reduces this gauge field system to the model (1).