STOCHASTIC QUANTIZATION AND SUPERSYMMETRY

by

R. KIRSCHNER
Sektion Physik
Karl-Marx-Universität Leibzig, GDR

Since the paper by Parisi and Wu [1] there is increasing interest in stochastic quantization.

The convergence of the stochastic relaxation process in the large-time limit to the corresponding quantum theory can be shown using the Fokker-Planck equation [2-5]. In the framework of perturbation theory the validity of stochastic quantization has been shown analyzing the stochastic graphs generated by the Langevin equation [6-8].

Consider a system with the action

\[ S = \int d^N x \, L(\phi(x)) \]  \quad (1)

and couple it to a white-noise random force \( \eta(x,t) \). The behaviour of the system is described by the Langevin equation with initial conditions at \( t = 0 \). We extend the range in time to the full axis defining, that for \( t < 0 \) a process takes place obtained from the process at \( t > 0 \) by time reflection.

The generating functional \( Z(j,t) \) of the stochastic equal-time correlation functions can be written as the

\[ + \text{Seminar given at the XXIII. Internationale Universitätswochen für Kernphysik, Schladming, Austria, February 20- March 1, 1984.} \]
generating functional of a quantum superfield theory \[8\] on the superspace consisting of \(n\) space dimensions, the additional time dimension \(t\) and the Grassmann dimensions \(\theta\) and \(\bar{\theta}\) with a restricted form of the current

\[
J_t(x,t',\theta,\bar{\theta}) = j(x) \delta(t-t') \delta(\bar{\theta}) \delta(\theta) .
\]  

(2)

The superfield Lagrangian is given by

\[
L(\Phi) = L_K(\Phi) + i L(\Phi),
\]

\[
L_K(\Phi) = \Phi (\partial_\theta \partial_{\bar{\theta}} - \frac{i}{2} \text{sgn } t (\partial_t \theta \partial_{\bar{\theta}} - \partial_{\bar{\theta}} \theta \partial_t)) \Phi .
\]  

(3)

\(\phi(x,t)\) is the lowest component of the superfield \(\Phi\). This superfield theory has a supersymmetry of the type of supersymmetric quantum mechanics (transforming \(t, \theta\) and \(\bar{\theta}\)).

The generating functional \(Z(j,t)\) is invariant with respect to a further supertransformation. This transformation leaves the following combination of the superfield coordinates invariant:

\[
\tau = t - \frac{i}{2} \text{sgn } t \bar{\theta} \theta .
\]  

(4)

We call this symmetry reduction supersymmetry. It is not a symmetry of the superfield Lagrangian. The invariance of \(Z(j,t)\) under this transformation is due to the special form of the current \(J_t\), eq. (2). The variation of the Lagrangian can be compensated by a change of variables in the functional integral.

Applying a time translation by an amount proportional to \(\bar{\theta} \theta\) it can be shown that the reduction supersymmetry coincides with the supertransformation invariance found in[7] by analyzing the stochastic graphs.

Besides of the stochastic quantization by relaxation processes discussed so far there is a similar quantization