Chapter 8
Local Linear Regression for Functional Predictor and Scalar Response

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Abstract The aim of this work is to introduce a new nonparametric regression technique in the context of functional covariate and scalar response. We propose a local linear regression estimator and study its asymptotic behaviour. Its finite-sample performance is compared with a Nadayara-Watson type kernel regression estimator via a Monte Carlo study and the analysis of two real data sets.

8.1 Introduction

There is nowadays a large number of fields where functional data are collected: environmetrics, medicine, finance, pattern recognition, ... This has led to the extension of finite dimensional statistical techniques to the infinite dimensional data setting. A classical statistical problem is that of regression: studying the relationship between two observed variables with the aim to predict the value of the response variable when a new value of the auxiliary one is observed.

In this work we consider the regression problem with functional auxiliary variable $X$ taking values in $L^2[0,1]$ and scalar response $Y$. A sample of random elements $(X_i, Y_i), 1 \leq i \leq n$, is observed, where the $X_i$ are independent and identically distributed as $X$ and only recorded on an equispaced grid $t_0, t_1, \ldots, t_N$ of $[0,1]$ whose internodal space is $w = 1/N$. It is assumed that the response variable $Y$ has been generated as

\begin{align*}
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\[ Y_i = m(X_i) + \varepsilon_i, \quad i = 1, \ldots, n \]  

(8.1)

and that the errors \( \varepsilon_i \) are independent, with zero mean and finite variance \( \sigma^2_{\varepsilon} \), and are also independent from any of the \( X_j \).

In the context of regression with functional data a common assumption is that \( m(x) \) is a linear function of \( x \). The linear model has been studied in a large number of works: see, e.g., Cardot, Ferraty and Sarda (2003), Ramsay and Silverman (2005), Cai and Hall (2006) and Hall and Horowitz (2007). Extensions of this model have been considered, for instance, by James (2002), Ferré and Yao (2003), Cardot and Sarda (2005) or Müller and Stadtmüller (2005). However, when dealing with functional data, it is difficult to gain an intuition on whether the linear model is adequate at all or which is the parametric model that would best fit the data, since graphical techniques are of scarce use here.

Here we are interested in estimating the regression function \( m \) in a nonparametric fashion. This problem has already been considered, for instance, by Ferraty and Vieu (2006), who study a kernel estimator of Nadaraya-Watson type

\[ \hat{m}_K(x) := \frac{\sum_{i=1}^{n} Y_i K_h(\|X_i - x\|)}{\sum_{i=1}^{n} K_h(\|X_i - x\|)}, \]  

(8.2)

where \( K_h(\cdot) := h^{-1} K(\cdot / h) \), \( h = h_n \) is a positive smoothing parameter and \( \| \cdot \| \) denotes the \( L^2[0, 1] \) norm. From now on \( K \) is assumed to be an asymmetrical decreasing kernel function. Observe that the estimator \( \hat{m}_K(x) \) is the value of \( a \) minimizing the weighted squared error

\[ \text{WSE}_0(x) = \sum_{i=1}^{n} (Y_i - a)^2 K_h(\|X_i - x\|). \]

Thus the kernel estimator given by (8.2) is locally approximating \( m \) by a constant (a zero-degree polynomial). However, in the context of nonparametric regression with finite-dimensional auxiliary variables, local polynomial smoothing has become the “golden standard” (see Fan 1992, Fan and Marron 1993, Wand and Jones 1995). Local polynomial smoothing at a point \( x \) fits a polynomial to the pairs \((X_i, Y_i)\) for those \( X_i \) falling in a neighbourhood of \( x \) determined by a smoothing parameter \( h \). In particular, the local linear regression estimator locally fits a polynomial of degree one. Here we plan to extend the ideas of local linear smoothing to the functional data setting, giving a first answer to the open question 5 in Ferraty and Vieu (2006): “How can the local polynomial ideas be adapted to infinite dimensional settings?”