The Classical Risk Model with Constant Interest and Threshold Strategy

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Abstract. In recent years, insurance risk models with dividend payments have been studied extensively. The threshold dividend strategy assumes that dividends are paid out at the maximal admissible rate whenever the surplus exceeds a certain threshold. In this paper, we consider the classical risk model with constant interest under the threshold strategy. We derive integro-differential equations for the expected discounted penalty function. In some special cases with exponential claims, we are able to obtain closed-form expressions for the expected discounted penalty function.

Keywords: classical risk model, dividend payments, threshold strategy

1 Introduction

Suppose that the surplus process of an insurer follows the classical risk model given by

\[ U(t) = u + ct - \sum_{k=1}^{N(t)} Z_k = u + ct - S(t), \quad t \geq 0, \tag{1} \]

where \( u \geq 0 \) is the initial surplus, \( c > 0 \) is the rate of premium, \( N(t) \) is a Poisson process with intensity \( \lambda > 0 \), and \( \{Z_k, k = 1, 2, \cdots\} \) is a sequence of independent and identically distributed non-negative random variables with common distribution \( F \). It is assumed that \( F(0) = 0 \) and that \( N(t) \) and \( Z_k \)'s are independent. Since \( N(t) \) indicates the number of claims up to time \( t \) and \( Z_k \)'s represent the claim amounts, the compound Poisson process \( S(t) \) is usually called the aggregate claims process. Note that the surplus process \( U(t) \) is also known as the compound Poisson risk model.

In the classical risk model \( (1) \), the assumption that the claim amounts \( Z_k \)'s are independent is often not met in many non-life insurance problems. In view of this, the study of risk models with various dependence relations among claim amounts as well as different classes of insurance business has become one of the popular actuarial topics in the past decade. Hence, one may try to extend the main results presented in this paper to a risk model.
with both correlated claims and dividend payments. Undoubtedly, such an extension would be a challenging one.

Assume that the insurer pays out certain amount of his surplus as dividends to the policyholders according to some dividend strategy. Let $D(t)$ be the total dividends paid up to time $t$, and $X(t)$ be the resulting surplus of the insurer at time $t$. Thus,

$$X(t) = U(t) - D(t), \quad t \geq 0.$$  \hspace{1cm} (2)

Here, we also assume that the insurer receives interest from his surplus at a constant rate $\delta > 0$. Then, the surplus process (2) becomes

$$Y(t) = e^{\delta t} \left( u + \int_0^t e^{-\delta s} dX(s) \right) = e^{\delta t} \left( u + \int_0^t e^{-\delta s} (U(s) - D(s)) \right), \quad t \geq 0.$$  \hspace{1cm} (3)

In the actuarial literature, the issue of dividend strategies has received remarkable attention recently. The study of the optimal dividend problem goes back to De Finetti (1957). Due to its practical importance, much research on dividend-payment problems has been carried out for various surplus processes since then. For example, see Gerber and Shiu (2006), Lin and Pavlova (2006), Yuen et al. (2007, 2008a, 2008b), and references therein.

Under the threshold dividend strategy, dividends are paid at the maximal admissible rate $\alpha < c$ whenever the surplus is above the threshold level $b$, and that no dividends are paid whenever the surplus is below $b$. For the surplus process (2) under the threshold strategy, Gerber and Shiu (2006) examined the optimal dividend problems and derived a rule for deciding between plow-back and dividend payout while Lin and Pavlova (2006) derived and solved two integro-differential equations for the expected discounted penalty function. For the surplus process (3), Fang and Wu (2007) studied the optimal dividend problems under the threshold strategy.

In this paper, we extend the work of Fang and Wu (2007) to investigating the expected discounted penalty function which embraces many important actuarial functions. In Section 2, we derive integro-differential equations for the expected discounted penalty function. In Section 3, a few examples with closed-form expressions for the expected discounted penalty function are presented.

2 Expected discounted penalty function under the threshold strategy

Under the threshold strategy, the surplus process (3) can be rewritten as

$$dY(t) = \begin{cases} cd\, dt - dS(t) + \delta Y(t)\, dt, & \text{if } Y(t) < b, \\ (c - \alpha)dt - dS(t) + \delta Y(t)\, dt, & \text{if } Y(t) > b, \end{cases}$$  \hspace{1cm} (4)