This chapter deals with the QCSP on the basis of container groups. It is studied as an isolated problem here and functionally integrated into the BACAP in the next chapter. Crane scheduling for container groups has been introduced by Kim and Park (2004). As noted by Moccia et al. (2006), the original QCSP model provided by Kim and Park shows an inaccuracy regarding the detection of crane interference. Unfortunately, even reworked problem formulations still tolerate certain cases of crane interference. A corrected problem formulation and a heuristic solution method have been provided by Bierwirth and Meisel (2009). The model and the heuristic are presented in Sects. 7.1 and 7.2, respectively. In Sect. 7.3 the QCSP is extended by incorporation of time windows for the cranes. Necessary modifications of the mathematical formulation and the solution method are described. Computational tests follow in Sect. 7.4. The study on the QCSP is concluded in Sect. 7.5.

7.1 Modeling the QCSP

7.1.1 Problem Description and Assumptions

In the QCSP for container groups a set of tasks $\Omega = \{1, 2, \ldots, n\}$ and a set of QCs $Q = \{1, 2, \ldots, q\}$ are given. Each task $i \in \Omega$ represents a loading or unloading operation of a certain container group. The tasks have individual processing times $p_i$ and bay positions $l_i$. Additionally, dummy tasks 0 and $T = n + 1$ with processing times $p_0 = p_T = 0$ are given to indicate the begin and the end of the service of the vessel. Further task sets are defined by $\Omega^0 = \Omega \cup \{0\}$, $\Omega^T = \Omega \cup \{T\}$, and $\Omega^t = \Omega \cup \{0, T\}$. Precedence relations may exist between pairs of tasks that are located within the same bay. Let $\Phi$ denote the set of precedence constrained task pairs. Furthermore, let $\Psi \supseteq \Phi$ denote the set of all task pairs for which it is known in advance that
they cannot be processed simultaneously. For each crane \( k \in Q \) a ready time \( r^k \) and an initial bay position \( l^k_0 \) is given. Without loss of generality, it is assumed that the cranes are indexed sequentially according to their initial positioning alongside the vessel. All QCs can move between two adjacent bays in an identical travel time \( t > 0 \). It is supposed that no two QCs can operate at the same bay at the same time. Moreover, cranes are not allowed to cross each other and have to keep a safety margin \( \delta \), measured in units of bays. The problem is to find completion times \( c_i \) for all tasks \( i \in \Omega \) on the cranes with respect to the constraints, such that the completion time \( c_T \) of the final task \( T \) (i.e., the makespan) is minimized.

Assumptions of the QCSP with container groups are:

1. Container groups are predefined from given stowage plans.
2. Processing of tasks is non-preemptive.
3. All QCs show an identical, deterministic transshipment productivity. For this reason fixed processing times of tasks are given. No consideration of individual container moves or crane cycle times is necessary.
4. The order of processing the tasks of a bay is completely determined by precedence constraints.
5. Crane operations cannot lead to an instable load configuration of the vessel, i.e., stability issues are not considered in the QCSP.
6. There is sufficient space beside the vessel to place idle QCs outside of the vessel area.

The described problem corresponds to a minimum makespan scheduling problem with parallel identical machines and precedence constraints. This problem is known to be \( \mathcal{NP} \)-hard in the strong sense, provided that more than two machines, non-preemption or non-uniform processing times are given, see Pinedo (2002).

### 7.1.2 Conventional Formulation of Interference Constraints

In the BACAP study, the productivity loss caused by crane interference has been modeled using an interference exponent \( \alpha \). Within the QCSP, interference effects are considered in more detail in order to generate feasible QC schedules.

In correspondence with models in machine scheduling, it is supposed that no two QCs can operate at the same bay at the same time. Moreover, since QCs are rail mounted, two types of interference constraints have to be respected:

- Non-crossing constraint: QCs cannot cross each other.
- Safety constraint: Adjacent QCs have to keep a safety margin at all times.

The safety margin \( \delta \) signifies a certain number of in-between bays between adjacent QCs. If, for example, \( \delta = 1 \) this means that QCs can simultaneously operate at the vessel if they are separated by at least one bay. Safety margin must be respected not only while QCs are working but also during the movement operations.