Chapter 5
A New Nonparametric Test of Symmetry

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Abstract We present a new nonparametric test for symmetry about an unknown location and investigate its large sample properties. Asymptotic normality of the test statistic is established and an estimator of the asymptotic variance is also presented. Results of a simulation study and data analysis are also presented.

5.1 Introduction

In many practical situations, it is important to determine whether the underlying population has a symmetric distribution. For example, if one is interested in estimating the measure of location, having a skewed distribution would give rise to consideration of more than one such measure. In the case of a paired sample problem with “treatment” and “control”, the hypothesis of “no treatment effect” implies that the paired difference \( Z = X - Y \) is symmetric about 0. Many robust statistical methods (see [35]) depend on the assumption of symmetry. In case symmetry is not valid, one would need to determine a symmetrizing transformation before applying the statistical procedures. Koziol [40] mentions using a test for symmetry as a screening procedure before applying the modulus family of transformations introduced by [36] for bringing data to be closer to normality.

The problem of testing univariate symmetry has received attention in the literature for quite some time. Past work in this area can be broadly classified into two groups – one where the center of symmetry is known (without loss of generality, this is then taken to be 0) and the other case is where the center of symmetry is completely unknown. A common class of nonparametric tests in the first category is the weighted sign test, which has been studied in detail by many authors, such as [29, 30, 37, 59]. Special cases of the former include

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the popular sign test and the signed-rank test. Other examples include works by \cite{14,20,47,53,56}. Aki \cite{3} proposed a new test of symmetry which generalized the results of \cite{14} and \cite{53}. Ahmad \cite{1} proposed asymptotically normal estimates of symmetry statistic by \cite{53}. Reynolds \cite{51} and Rosenstein \cite{52} developed a sequential signed-rank test for symmetry about a known value. Further work have been done by \cite{6,8,9,17,21,31–33,39,41,42}. Some of the recent work in this area have been by \cite{2,15,45,46,58}.

The case of unknown center of symmetry is a more realistic and practical problem. One of the earliest articles in this area is by \cite{34} where the authors use the difference of the mean and median as a measure of skewness. One group of tests is obtained by applying the tests with known center, where the center is estimated from the data. Gastwirth \cite{26} proposed the modified sign test, whereby the unknown point of symmetry is estimated by the sample mean and a sign test is developed for symmetry around this point. Other articles in this category include those by \cite{10,21,28}. Boos \cite{13} propose a test for symmetry based on the Hodges–Lehman estimator of location. Davis and Quade \cite{19} and Randles \cite{50} propose asymptotically distribution free tests of symmetry. Antille and Kersting \cite{5}, Antille et al. \cite{6} and Finch \cite{25} propose tests based on symmetric differences of spacings. Csörgő and Heathcote \cite{18} and Koutrouvelis \cite{38} use the empirical characteristic function to develop tests for symmetry about an unknown location, that do not require estimation of the unknown location. Koziol \cite{40} propose using rank-based tests of symmetry with known center where the center is estimated using asymptotically efficient methods. Eubank et al. \cite{23} also propose a class of rank-based tests of symmetry. Schuster and Barker \cite{54} proposed a symmetric bootstrap procedure for testing symmetry about unknown center, which was later examined in further detail by \cite{7} and extended to the dependent case by \cite{48}. Cabilio and Masaro \cite{16} propose a test of symmetry about an unknown location based on deviation of the sample mean from the sample median, similar to the lines of \cite{34}. Some of the recent work in this area are \cite{22,43,44}.

In this article, we develop a new test of symmetry for univariate distributions when the center of symmetry is unknown. Our test is based on a new characterization of symmetry. This article proceeds as follows. In Sect. 5.2, we propose the new estimator and investigate its properties. In Sect. 5.3, we provide results of simulation studies in support of the theoretical properties. In Sect. 5.4, we provide a real-life example of use of the proposed statistic. Finally, we end with concluding remarks in Sect. 5.5. The proofs of results presented here are presented in the Appendix.

5.2 Theory

Suppose \(X_1, \ldots, X_n\) is a set of observations (not necessarily i.i.d.) from an underlying continuous distribution with cdf \(F\) and corresponding pdf \(f\). We want to test for symmetry of \(f\) about an unknown point \(\mu\) (say). For any \(p \in (0, 1)\), consider the function