4 Distributed Spatial Reasoning

It is quality rather than quantity that matters.

Lucius Annaeus Seneca, c. 4 BC - 65 AD

Reasoning about perceived context information is fundamental for autonomous systems to draw conclusions about their spatial contexts and adapt to changing situations at runtime. According to our focus on qualitative spatial relations, this chapter builds on the qualitative abstractions presented in Chapter 3 and is concerned with inferring new from known relations. In Section 4.1, we first give an overview of approaches for qualitative spatial reasoning, with a focus on binary relations and their properties as well as compositional reasoning about them. In Section 4.2, we discuss reasoning about positional and directional relations in more detail and present composition tables for distance and orientation relations. The main part of this chapter is about an algorithm for inferring and distributing spatial relationships among autonomous digital artifacts over multiple hops, which is presented in Section 4.3 and evaluated by simulation means in Section 4.4. The general idea is that artifacts recognize spatial relations to others in their vicinity by sharing spatial context information, and infer relations to artifacts out of communication range (i.e. data cannot be exchanged directly, but just via other artifacts) by means of qualitative spatial reasoning techniques which are discussed in this chapter. In particular, the algorithm makes use of composition tables or the properties of binary relations in order to provide artifacts, which are out of communication range, with an awareness about their spatial relationships to each other. Section 4.5 eventually discusses benefits and drawbacks of the presented qualitative spatial reasoning approaches and elaborates on their use for autonomous embedded systems.

4.1 Overview of Qualitative Approaches

Qualitative spatial reasoning in general builds upon spatial relations as discussed in Section 3.2, and aims at providing computers with the ability to draw conclusions in a qualitative manner. With regard to our focus on autonomous embedded
Distributed Spatial Reasoning

systems, qualitative spatial reasoning allows digital artifacts to infer new relations based on known ones, and thus improves their spatial awareness in that implicitly available knowledge is made explicit and can be directly used at the application level. It is typically realized with calculi over sets of qualitative spatial relations, from which a considerable number has been developed within the past two decades. They focus on different spatial properties, where most of the research efforts have been concerned with topological reasoning about regions as well as reasoning about distance and orientation relations on points or line segments [DFWW06, KB99]. In the following, the foundations of qualitative spatial reasoning will be discussed as far as they are relevant to understand the proposed approach for relationship distribution presented in Section 4.3; starting points for further information on this research field can be found in several survey papers [CH01, FR93, HN02] and books [For97, Her94].

As stated in Section 3.2.2, this thesis is concerned with binary relations between artifacts – or, more precisely, between Zones-of-Influence which are associated with them; we use object synonymously for Zone-of-Influence in the following, as it is the common term in the research field of qualitative spatial representation and reasoning. A relation $R$ between two objects $x$ and $y$ (i.e. $(x,y) \in R$) is often denoted as $R(x,y)$, where $x$ is referred to as the primary and $y$ as the reference object. According to [DFWW06], a binary spatial calculus consists of (i) a domain $D$ containing the spatial primitives (e.g. points in two-dimensional space representing physical objects), (ii) a finite set $BR$ of typically jointly exhaustive and pairwise disjoint 2-ary base relations on that domain, as well as (iii) a set of operations which are closed over the powerset $R$ of these base relations. Thus, the result of an operation may be a compound relation (i.e. the union of multiple base relations), which allows for expressing uncertainty (e.g. that an object $x$ is either left or front of $y$); for this reason, the operations of a calculus have to be defined for all possible unions of base relations (cf. Section 4.2.2). A special case in this regard is the universal relation, which is a disjunction of all base relations and represents the complete lack of knowledge about the relation between the respective two objects [NS02, WFDW07].

The following operations union ($\cup$), intersection ($\cap$) and composition ($\circ$) on binary relations are relevant for our approach, where $R, S \in R$ [DFWW06]:

\[
\begin{align*}
R \cup S &= \{x | (x \in R) \lor (x \in S) \} & (4.1) \\
R \cap S &= \{x | (x \in R) \land (x \in S) \} & (4.2) \\
R \circ S &= \{(x,z) | \exists y \in D : (x,y) \in R \land (y,z) \in S \} & (4.3)
\end{align*}
\]