Economies of Scale in Hub & Spoke Network Design Models: We Have It All Wrong

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1 Traditional Models for Hub & Spoke Network Design

The hub & spoke network design problem is a strategic logistics planning problem with applications for airlines, telecommunication companies, computer networks, postal services, and trucking companies, for example. Basically, the problem in all these applications is that for a given set $V = \{1, \ldots, n\}$ of nodes (airports, computers, post offices, depots, ...) goods must be transported between possibly every pair of nodes. Direct connections between every pair of nodes would result in $n(n-1)$ linkages which is impractically high and economically non–profitable. Consider, for instance, an airline that serves several airports worldwide. Offering non–stop flights between every pair of airports would require a huge amount of planes and crews and many empty seats on board could be observed for many connections. In such settings, it turns out to be reasonable to install one or more so–called hub locations where direct links are then available to hub nodes as indicated in figure 1 where nodes 3, 6, and 9 are assumed to be hubs. Transporting goods from, say, node 1 to node 11, can then be done via hubs 3 and 6.

Roughly speaking, the network design problem at hand can be couched as follows: Given a graph with node set V and edge set $E = V \times V$, select one or more nodes from V to become hub nodes and select some edges from E to become transportation links. For each pair of nodes $(i, j) \in V \times V$ we have a quantity $q_{ij} \in \boldsymbol{R}_{\geq 0}$ that is to be transported from node i to node j. Established models assume that the unit cost of transportation using an edge e is $c_e \in \boldsymbol{R}_{\geq 0}$ and that, if e connects two hub nodes, a discount can be gained such that the unit cost of transportation using edge e is $\alpha c_e \in \boldsymbol{R}_{\geq 0}$ with $0 \leq \alpha \leq 1$. We will question these cost assumptions in section 3 and discuss alternatives. Note that the costs may be asymmetric, i.e. $c_{ij} = c_{ji}$ may not be true.

For the hub & spoke network structure one can wish to have specific characteristics that define the design problem to be solved. Some usual and

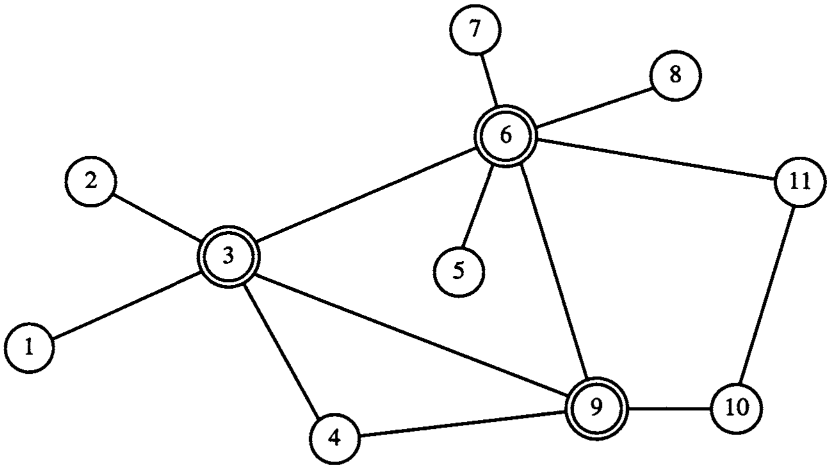

Figure 1: An Illustration of a Hub & Spoke System

basic features are the following:

- What determines the number of hubs?
  - Hub location problems with fixed hub costs: Installing a hub at node h incurs a fixed cost $f_h \in \boldsymbol{R}_{\geq 0}$. The number of hubs is a result of the planning process.
  - p–Hub median problems: The number p of hubs is predefined. Fixed hub costs are usually assumed to be the same for all nodes so that they can be ignored for the purpose of optimization.
- How are the non–hub nodes connected?
  - Single allocation: Each non–hub node is allocated to a unique hub. In a single allocation network, node 4 (figure 1) would not be allowed to have a direct link to two hubs.
  - Multiple allocation: Non–hub nodes (like node 4 in figure 1) may be connected to several hubs.
  - Direct services: Non–hub nodes may have a direct connection like nodes 10 and 11 in figure 1.

Throughout this paper, we assume that the set of hubs is fully meshed, i.e. the subgraph induced by the hub nodes is complete, and that the objective for designing the network is to minimize the sum of relevant costs.