Using Homogeneous Groupings in Portfolio Management

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Abstract. Often, in situations of uncertainty in portfolio management, it is difficult to apply the numerical methods based on the linearity principle. When this happens it is possible to use nonnumeric techniques to assess the situations with a non linear attitude. One of the concepts that can be used in these situations is the concept of grouping.

In the last thirty years, several studies have tried to give good solutions to the problems of homogeneous groupings. For example, we could mention the Pichat algorithm, the affinities algorithms and several studies developed by the authors of this work.

In this paper, we use some topological axioms in order to develop an algorithm that is able to reduce the number of elements of the power sets of the related sets by connecting them to the sets that form the topologies. We will apply this algorithm in the grouping of titles listed in the Stock Exchange or in its dual perspective.

1 The Two Perspectives for Topological Fuzzification in Economy

It is well known that a topology $E$ in uncertainty can be defined by the subset $T(E)$ of the opened that accomplishes the following axioms (Chang, 1968). Note that for further reading on fuzzy topology and pretopology, we recommend, for example (Badard, 1981; Bayoumi, 2005; Du et. al., 2005; Fang and Chen, 2007; Fang and

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where $A_j, A_k$ may have a different meaning depending on the criteria used for the fuzzification. In the first case, they are fuzzy subsets of the referential set $E$ that accomplishes the previous axioms, and in the other case, they are elements of the power set established from a referential set $E$ of fuzzy subsets. In the first case, the referential set $E$ is formed initially by the elements of the referential set of the fuzzy subsets. In the second case, their elements are the fuzzy subsets themselves. As it has been pointed out in other works (Gil-Aluja, 2003; Gil-Aluja and Gil-Lafuente, 2007) the selection of one of these perspectives depends mainly on the objectives of the analysis.

In an economical and financial context, we consider that it is relevant to think about the meaning of the components of both cases. For doing this, we will use the representability of the notion of fuzzy subset. The reason is because for an economist, a fuzzy subset is a descriptor of a physical or mental object; and this description is developed by putting different levels to the elements of the referential set formed by the attributes of the objects that we want to describe. Then, in the economic environment it is possible to accept that in the first case, the referential set $E$ is formed by the set of attributes that describe each object while in the second case, the referential set $E$ is formed by the fuzzy subsets, where each of them describe an object.

If we consider financial products such as titles listed in the Stock Exchange, the description of each of them will take place by a certain number of attributes such as the expected rentability, the liquidity capacity without loses, etc., all of them classified at certain level. In this assumption, the referential set $E$ will be formed in the first case by the expected rentability, the liquidity capacity, etc., and in the second case, by the different titles listed in the Stock Exchange.

With this approach, the $A_j, A_k \in T(E)$, the elements of the open set $T(E)$, are in the first case, fuzzy subsets with the referential of their attributes and in the second case, fuzzy subsets or groupings of fuzzy subsets with the same referential.

It is obvious that the concept of economic representability is different in each case. Then, the Axioms 1 and 2 acquire the following meaning:

- In the first case, Axiom 1 shows that the fuzzy subset (title listed in the Stock Exchange) with a null level in all its attributes is an open set and so is (Axiom 2) the fuzzy subset (title listed in the Stock Exchange) with level one (maximum) in all its attributes.
- In the second case, Axiom 1 shows that in a situation without fuzzy subsets we have an open set. In this case, the set of all the fuzzy sets (all the titles listed in the Stock Exchange) is also an open set (Axiom 2).