Giuseppe Peano and Mathematical Analysis in Italy

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If you enter what was latterly the office of the Head of the former *Istituto di Analisi Matematica* of the University of Torino, you find before you a group of photographs which recall those Professors who held the Chair of Analisi (or *Calcolo*) in the period from 1811 to 1972; among them, side by side with Giovanni Plana (who occupied the Chair from 1811 to 1864), Angelo Genocchi (from 1864 to 1889), Enrico D'Ovidio (from 1872 to 1918), is the mild countenance of Giuseppe Peano, Professor of Calculus from 1890 to 1931; next to him is his successor Francesco G. Tricomi, Professor of Analysis from 1925 to 1972.

Peano, in fact, who left the mark of his brilliant mind and results in a variety of fields of knowledge, started from Analysis as a teacher and scientist. Graduating in 1880, he spent a year as assistant of D'Ovidio, then was assistant of Genocchi from 1881 to 1890; *Libro docente di Calcolo infinitesimale* in 1884, he became a full professor of Infinitesimal Calculus in December 1890 and remained in this role until 1931; he subsequently transferred to the Chair of *Matematiche complementari* in the last year of his life (1931/32).

1.1 Peano and Classical Analysis

It is his interest in Analysis that seems really to have been the basis on which were to develop the various fields of research which made clear the characteristic originality, the perspicacity, the independence of Peano’s thought: fields of research that ranged from Analysis to Geometry, to numerical Calculus, to Logic, to History, and finally to Linguistics, to say nothing of his many other studies, some of which were of a practical or social origin.

In his first years as a University teacher, teaching itself had a crucial influence on his research, leading him to clarify and go more deeply into the various topics of the Calculus courses, thanks to his inborn, exceptional critical spirit as well as his vast classical culture.
The decisive opportunity for a profound re-examination of Analysis was offered to him, in 1884, by the writing of the text “A. Genocchi, *Calcolo differenziale e principii di calcolo integrale, pubblicato con aggiunte da G. Peano*” (Peano 1884c), which originated from Genocchi’s lectures. In reality these lectures were greatly enriched by the addition of *Annotazioni (Notes)*, which are the most original part of the book and which, as Genocchi himself chose to make clear, almost as though he wanted to keep his distance from them, were entirely the work of the young Peano: here were critical observations, here, often by means of brilliant counter-examples, were stressed – and then corrected simply and rigorously – inexactitudes frequently repeated and included in most texts of the time. Peano himself specified, in the commemoration (Peano 1890a) of his Master, Genocchi:

Ill at that time, he wished to remain extraneous to the whole undertaking. Making use of summaries made by students during his lectures, I compared them point by point with all the main treatises on calculus, and with original papers, thus taking into account the work of many. I consequently made many additions to his lectures, and some modifications.1

The book was also published in a German edition (Bohlman, Schepp 1899t).

Meanwhile, the characteristics of his keen mind and the scrupulousness, typical of a careful scientist, with which he enriched his knowledge of the developments of contemporary mathematics made it possible for him to perceive the need for rigour and abstract thought which were beginning to be developed and to spread in the mathematical environment, thanks to the works of a number of writers among whom we may mention – in Italy – Ulisse Dini, Salvatore Pincherle, Cesare Arzelà. The new spirit that was animating the Analysts in those years, “heroic” for the evolution of Analysis after the construction of the theory of continuous functions by Lagrange and Cauchy, is clearly perceptible in the Introduction to Dini’s treatise *Fondamenti per la teorica delle funzioni di variabili reali* (Dini 1878), where the author states:

I shall be happy if […] it contributes to make known certain remarks and results which in recent times have shaken the foundational principles of Analysis, only to rebuild them immediately on more solid bases.2

Peano’s brilliant results, which are still currently studied in the first two years of our courses in Analysis and which, as we have seen, sprang up on the margins of his teaching activities in the last twenty years of the 19th century, fit in with this critical spirit: they are expounded, not only in some printed scientific papers and in the

1 G. Peano (1890a), 198–199: “Egli, allora malato, volle rimanere estraneo a tutto il lavoro. Io, servendomi di sunti fatti da allievi alle sue lezioni, li paragonai punto per punto con tutti i principali trattati di calcolo, e con Memorie originali, tenendo così conto dei lavori di molti. Feci in conseguenza alle sue lezioni molte aggiunte, e qualche modificazione.”

2 U. Dini (1878), viii: “Sarò lieto se […] esso contribuirà a rendere familiari alcune osservazioni e alcuni risultati che in questi ultimi tempi hanno scosso per poi riedificare, immediatamente, su basi più solide i principii fondamentali dell’Analisi.”