Adaptive numerical flow simulation

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Summary. The paper is concerned with three-dimensional adaptation procedures for the numerical solution of compressible flow problems. An algorithm is devised for steady state transonic flow with strong shocks. Original three-dimensional mesh refinement indicators are proposed. The grid alignment is controlled by the anisotropic mesh adaptation (AMA) strategy. An original smoothing procedure in the minimization of the interpolation error in AMA is applied. Computational results for the inviscid flow are presented.

1 Introduction

The cell-based upwinding finite volume schemes have been very successful in recent years in generating good numerical solutions to inviscid flow problems. For viscous flow problems, with small viscosity and heat conductivity coefficients, the convection effects dominate those of diffusion and therefore the discretization of the inviscid terms plays a principal role. Here we present an adaptation procedure for the 3D Euler equations which can be extended to the solution of the Navier–Stokes equations in the frame of the combined finite volume–finite element method [8].

2 Formulation of the problem

We consider gas flow in a space-time cylinder \( Q_T = \Omega \times (0, T) \), where \( \Omega \subset \mathbb{R}^d, d = 3 \), is a bounded polyhedral domain representing the region occupied by the fluid and \( T > 0 \).

The complete system of viscous \( d \)-dimensional compressible flow consists of \( N \) (\( N = d + 2 \)) equations: the continuity equation, Navier–Stokes equations and energy equation. It can be written in the form

\[
\frac{\partial w}{\partial t} + \sum_{s=1}^{d} \frac{\partial f_s(w)}{\partial x_s} = \sum_{s=1}^{d} \frac{\partial R_s(w, \nabla w)}{\partial x_s} \quad \text{in} \ Q_T. \tag{1}
\]

Here

\[
w = (w_1, w_2, \ldots, w_N)^T = (\rho, \rho v_1, \ldots, \rho v_d, e)^T,
\]

\[
w = w(x, t), \quad x = (x_1, \ldots, x_d) \in \Omega, \ t \in (0, T)
\]

and \( f_s(w), R_s(w, \nabla w) \) are inviscid (Euler) and viscous fluxes, respectively. For their form see [4]. We use standard notation: \( t \) – time, \( x_1, \ldots, x_d \) – Cartesian coordinates in \( \mathbb{R}^d \), \( \rho \) – density, \( v = (v_1, \ldots, v_d) \) – velocity vector with components \( v_s \) in the
directions $x_s$, $s = 1, \ldots, d$, $e$ – total energy. We neglect outer volume force and assume that the state equation for a perfect gas is used. The functions $f_s$ are defined in the set $D = \{(w_1, \ldots, w_N) \in \mathbb{R}^N; w_1 > 0, w_N - \sum_{i=2}^{N-1} w_i^2/(2w_1) > 0\}$. The viscous terms $R_s$ are obviously defined in $D \times \mathbb{R}^2$.

The above system is equipped with initial and boundary conditions. For their definition see [5]. The complete system (1) is split into inviscid and viscous parts:

$$\frac{\partial w}{\partial t} + \sum_{s=1}^{d} \frac{\partial f_s(w)}{\partial x_s} = 0,$$

$$\frac{\partial w}{\partial t} = \sum_{s=1}^{d} \frac{\partial R_s(w, \nabla w)}{\partial x_s},$$

which are discretized separately. In this paper we concentrate on the adaptive algorithm for the finite volume solution of system (2). This is the main part of the iterative algorithm for the solution of system (1) via the splitting (2), (3), described in [8].

3 Data structure

The discretization of system (2), based on mathematical properties, is carried out with the use the finite volume method (FVM) which is very popular because of its flexibility and applicability and because it reflects well important characteristic features of compressible flow, e.g., the fact that discontinuities arise in the weak solution. We are mainly interested in the precise computation of shock waves, which are hypersurfaces characterized by discontinuities in velocity (and possibly in other quantities) and by nonzero mass flux of the fluid through elements of the shock wave. The geometrical data structure is represented by the tetrahedral mesh $\mathcal{D}$, which makes possible sufficiently precise discretization of the computational domain and adaptive remeshing during the computational process. The numerical solution is assumed to be a piecewise constant finite volume function. In practical computations we try to achieve sufficiently accurate solutions with a precise resolution of shock waves and therefore use an adaptive refinement of the mesh. As the position and form of shock waves are not known a priori, it is convenient to use adaptive algorithms using information from the previously computed approximate solution.

The most important tool in an adaptive method is the choice of a suitable error indicator and, for flows with shocks, a shock indicator as well. First we are concerned with an adaptive shock-capturing algorithm employing a divided differences approach and only indicating admissible shock waves.

3.1 Shock indicator

Let $\mathcal{W}$ be the shock wave. (Recall that its position is not known a priori.) From the Rankin–Hugoniot conditions on the shock wave $\mathcal{W}$ and from the second law of thermodynamics, using the relation between the Hugoniot shock adiabatic function