Thin-walled beams

Typical aeronautical structures involve light-weight, thin-walled, beam-like structures that must operate in a complex loading environment where combined axial, bending, shearing, and torsional loads are present. These structures may consist of closed or open sections, or a combination of both. A closed cross-section is one for which the thin wall forms one or more closed paths; in the opposite case, it is an open section. This distinction has profound implications for the structural response of the beam, most importantly when it comes to shearing and torsion.

In the analysis of thin-walled beams, the specific geometric nature of the beam consisting of an assembly of thin sheets will be exploited to simplify the problem’s formulation and solution process. Figures 8.1 to 8.4 show different types of thin-walled cross-sections. Figure 8.1 shows a beam with a closed section, as opposed to the open section of fig. 8.2. A combination of both types depicted in fig. 8.3 is also possible. Finally, multi-cellular sections such as shown in fig. 8.4 are very common in aeronautical constructions.

8.1 Basic equations for thin-walled beams.

The geometry of the section is described by a curve, $C$, drawn along the mid-thickness of the wall, see figs. 8.1-8.4. A curvilinear variable, $s$, measuring length along this contour is defined with an arbitrary origin. This variable defines an orientation along $C$ at all points. Of course, this orientation can be chosen arbitrarily. The wall thickness, $t(s)$, can vary from point to point along the contour. For multi-cellular sections, a number of different curves are used to completely describe the section, and a corresponding number of curvilinear variables define the length and orientation of these various curves.

8.1.1 The thin wall assumption

In thin-walled beams, the wall thickness is assumed to be much smaller than the other representative dimensions of the cross-section. Considering fig. 8.1, this means
8.1.2 Stress flows

As discussed in sections 5.4.2 and 5.5.2, the stress components acting in the plane of the cross-section are assumed to be negligible as compared to other stress components. This implies that $\sigma_2 \ll \sigma_1$ and $\sigma_3 \ll \sigma_1$ and furthermore, $\tau_{23} \ll \tau_{12}$ and $\tau_{23} \ll \tau_{13}$; it is then assumed that the only non-vanishing stress components are the axial stress, $\sigma_1$, and the transverse shear stresses, $\tau_{12}$ and $\tau_{13}$.

Given the geometry of thin-walled beams described in the previous section, it is not convenient to work with the Cartesian components of transverse shear stress, rather, it is preferable to resolve the shear stress into its components parallel and normal to $\mathbf{C}$, denoted $\tau_s$ and $\tau_n$, respectively, as illustrated in fig. 8.5. The relationship between these two sets of stress components is

$$
\tau_n = \cos \alpha \tau_{12} + \sin \alpha \tau_{13} = \tau_{12} \frac{dx_3}{ds} - \tau_{13} \frac{dx_2}{ds}, \quad (8.2a)
$$

$$
\tau_s = -\sin \alpha \tau_{12} + \cos \alpha \tau_{13} = \tau_{12} \frac{dx_2}{ds} + \tau_{13} \frac{dx_3}{ds}. \quad (8.2b)
$$