This paper discusses Carnap’s attempts in the late 1920s to provide a formal reconstruction of modern axiomatics.\footnote{Carnap’s early contributions to axiomatics comprise two published articles, (Carnap 1930) and (Carnap & Bachmann 1936), his logic manual \textit{Abriss der Logistik} (Carnap 1929), and the posthumously published typescript \textit{Untersuchungen zur allgemeinen Axiomatik} (Carnap 2000).} One interpretive theme addressed in recent scholarly literature concerns Carnap’s underlying logicism in his philosophy of mathematics from that time, more specifically, his attempt to “reconcile” the logicist approach of reducing mathematics to logic with the formal axiomatic method. For instance, Awodey & Carus (2007) characterize Carnap’s manuscript \textit{Untersuchungen zur allgemeinen Axiomatik} from 1928 as a “large-scale project to reconcile axiomatic definitions with logicism, and transform implicit into explicit definitions.” (ibid., 29) It is argued that Carnap’s central idea was to balance a Fregean (or Russellian) foundational stance with the modern model-theoretic viewpoint introduced in Hilbert’s \textit{Grundlagen der Geometrie} (see (Reck 2004)). It was also shown in recent literature that Carnap’s attempt to provide a logicist reconstruction of axiomatics is limited in several ways.\footnote{Compare in particular (Awodey & Carus 2001), (Reck 2004), (Reck 2007), and (Bonk & Mosterin 2000).} No closer attention, however, has so far been dedicated to some of the details of his proposed reconciliation.

The aim in this paper is to give a closer analysis of Carnap’s theory of general axiomatics in \textit{Untersuchungen}, specifically of the impact of a tacit logicist assumption underlying his semantics for axiom systems. The central notion to be investigated in this respect is that of a \textit{logical interpretation} (or a \textit{logical model}). Carnap mentions the term several times in his writings from the time without specifying its exact meaning. His understanding of the logical interpretability of an axiom system will be examined in comparison with similar accounts in the work of Alfred Tarski and Friedrich Bachmann. The subsequent investigation is guided by the following interpretive questions:

1. In what sense is Carnap’s work on general axiomatics in \textit{Untersuchungen} entangled with classical logicism?
2. What is the specific role of the logical interpretability of axiom systems in his theory?

3. Given Carnap’s logicist background, does his formal reconstruction meet the semantic innovations of modern formal axiomatics?

The discussion of these points will start with a brief presentation of Carnap’s theory of general axiomatics and his definition of formal models (Section 2). Following this, the second question concerning his specific ties with logicism will be discussed in Section 3. In Section 4, it will be argued that Carnap’s attempt to reconstruct the model theory of axiomatic theories is based on a specific logicistic premise, namely a definability condition for formal models. The details of this assumption will be discussed in Section 5 following the comparison of two similar accounts by Tarski (Section 4.1) and Bachmann (Section 4.2). Finally, in Section 6, the third question will be addressed. As will be shown, Carnap’s logicist premise has strong limiting effects for his attempt to express the semantic metatheory of formal axiomatics.

CARNAP’S General Axiomatics IN 1928

Carnap’s theory of general axiomatics is presented in Untersuchungen (Carnap 2000) and, in published form, in the second part of Abriss der Logistik (Carnap 1929, 70-72). The basic idea expressed there is that axiomatics is a kind of “applied logicism,” i.e. an application of a simplified version of Russell’s logical type theory (henceforth STT). Carnap’s formal reconstruction is based on the distinction between two possible conceptions of axiomatics, viz. “contentual” (“inhaltliche”) and formal axiomatics. In the former, the “primitive symbols” (“Grundzeichen”) of a theory have a fixed meaning. Axioms and consequences of the theory make substantive claims about the logical relations of these interpreted terms. In the latter, the primitives are implicitly defined by the theory and do not have a fixed interpretation. This difference from contentual axiomatics is specified in the following passage in Untersuchungen:

One can also understand the primitive concepts as unspecified elements and as relations of an unspecified domain where it is only stipulated that they relate to each other as specified in the axioms. In case that elements and relations are found in different domains that satisfy these formal specifications, then the axiom system can be applied to each one of these domains; in each of these domains also the consequences of the axiom system hold under the respective interpretation. (Carnap 2000, 88)3

3 “Oder aber man fasst die Grundbegriffe auf als unbestimmte Gegenstände und Beziehungen eines unbestimmten Gebietes, von denen nur festgelegt wird, dass sie sich so zueinander verhalten, wie es in den Axiomen bestimmt wird. Finden sich auf verschiedenen Gebieten Gegenstände und Beziehungen, die diese formalen Bestimmungen erfüllen, so kann das Axiomensystem auf jedes dieser Gebiete bezogen