ERROR CONTROL, MESH UPDATING SCHEMES AND AUTOMATIC ADAPTIVE REMESHING FOR FINITE ELEMENT ANALYSIS OF UNSTEADY EXTRUSION PROCESSES

G.C. HUANG, Y.C. LIU and O.C. ZIENKIEWICZ
Institute for Numerical Methods in Engineering,
University College of Swansea, U.K.

Summary
Error estimation, adaptive remeshing as well as different special mesh updating schemes are applied in the finite element simulation of hot extrusions through shear die and tapered die. The material grid cooperated with the analytical mesh is introduced to simulate the real forming processes. The algorithms are presented.

1. Introduction
The flow formulation approach [1] and other large deformation finite element methods have made it possible to simulate the complicated forming processes. An intrinsic difficulty in the analysis is the constantly changing configuration of the deformed body. If simple updating is used, this results in a very much distorted mesh which either makes the further analysis impossible or introduces large approximation errors. These problems can be solved by remeshing or by using different mesh updating schemes, such as AEL (Arbitrary Eulerian Lagrangian) method [2,3], or by pseudo-concentration method [4]. The last two methods require a prior knowledge of the forming processes, therefore are not in general use. However, for some special cases they are simple and economical. Remeshing scheme generates a new mesh over the deformed domain with all the state variables transferred from the old mesh, therefore it has the merit in resuming the analysis and reducing the discretization error caused by the improper mesh. A simple error estimation and adaptive remeshing scheme was first suggested by Zienkiewicz and Zhu [6] for linear problems and later introduced in the forming analysis by Zienkiewicz, Liu and Huang [7]. The strategy of this method is to reduce the error to a specified level by using an adaptively generated mesh which is controlled by an error indicator. The degrees of freedom are kept as few as possible to achieve this aim.

In the present paper the error control and different mesh updating schemes are described. The concept of material grid and analytical mesh is introduced, which can be explained schematically by a family tree in Fig. 1. The analytical mesh is for the use of the finite element analysis, while the material grid, as mentioned by Cheng [5], is only for output display purposes and for comparisons with the experiments.

Six-noded triangular element with a single pressure and the nodal velocities is used for the analysis. The schemes are applied in some unsteady extrusion problems.

2. Error Estimation and Adaptive Remeshing Scheme

Error (only discretisation error is concerned) is defined in terms of the "energy" norm as

$$\| \varepsilon \|_S^2 = \int_{\Omega} (\hat{S} - \bar{S})^T (2\mu)^{-1} (\hat{S} - \bar{S}) \, d\Omega$$  \hspace{1cm} (1)

where $\hat{S}$ stands for deviatoric stress vector. $\bar{S}$ and $S$ are the exact solution and finite element solution respectively. $\mu$ is the viscosity obtained by finite element method. Since the exact solution of deviatoric stress is not available, a smoothed one, $\bar{S}$ (which is one polynomial order higher than $\hat{S}$) is used instead of $\bar{S}$. The least square method is adopted to obtain $\bar{S}$ by the following equation

$$\left[ \int_{\Omega} N^T N d\Omega \right] \bar{S}^* = \int_{\Omega} N^T \bar{S} d\Omega \quad \text{and} \quad \bar{S}^* = N\bar{S}^*$$ \hspace{1cm} (2)

$\bar{S}^*$ being the smoothed nodal deviatoric stress components. Now the predicted error norm can be written as,

$$\| \varepsilon \|_S = \int_{\Omega} (\bar{S} - \hat{S})^T (2\mu)^{-1} (\bar{S} - \hat{S}) \, d\Omega$$ \hspace{1cm} (3)

The criterion for the remeshing is set up as

$$\| \varepsilon \|_S / \| \varepsilon \|_S = \eta^0 \leq \bar{\eta} \quad \text{where} \quad \| \varepsilon \|_S = \int_{\Omega} (\bar{S}^* - \hat{S}^*)^T (2\mu)^{-1} \bar{S}^* d\Omega \hspace{1cm} \text{(4, 5)}$$

$\bar{\eta}$ is the prescribed percentage error. Whenever Eq. (4) is unsatisfied the remeshing is needed. The remeshing procedure is controlled by the aiming percentage error $\eta_{aim}$ which is the percentage error we desire to reach after remeshing. The adaptive remeshing scheme is based on the idea that after remeshing each element possesses the same error and the total percentage error is equal to the aiming percentage error $\eta_{aim}$, i.e.

$$\left( \| \varepsilon \|_S^* \right)_\text{after} / \| \varepsilon \|_S^2 = \eta_{aim}^{-2}$$ \hspace{1cm} (6)

Since the square of the total error norm is the sum of the local element contributions, i.e.

$$\| \varepsilon \|_S^2 = \sum_{i=1}^N \left( \| \varepsilon \|_S^2 \right)_i$$ \hspace{1cm} (7)

If each element has the same error, then the aiming local error

$$\left[ \left( \| \varepsilon \|_S^* \right)_i \right]_{\text{after}} = \| \varepsilon \|_S / \sqrt{N} \eta_{aim}$$ \hspace{1cm} (8)

Asymptotically the error is dependent on the size of the element, $h$ [7], i.e.

$$\left( \| \varepsilon \|_S^* \right)_i \propto h_i^p$$ \hspace{1cm} (9)