5 Location of Seismic Events

The location of seismic events is of fundamental importance, since all subsequent seismological processing depends, to some degree, upon the event position and the distances to the stations. Finding the location of a seismic event amounts to solving an inverse problem – a set of unknowns has to be retrieved from a set of data. The four unknowns consist of the location and origin time vector, \( x = (h, t_0)^T = (x_0, y_0, z_0, t_0)^T \). In the case of a mine seismic network, the unknowns are to be retrieved from data consisting of P and S wave arrival times, the direction of the wavefronts, both of which are derived from waveforms, the velocity model and the station coordinates. The data have associated errors which can produce location errors. The location errors also depend on the spatial distribution of stations with respect to the position of the source, discussed in Section 5.4 below, and on the physics of the rupture process. If slow rupture starts at a certain point, the closest station(s) may record waves radiated from that very point while others may only record waves generated later in the rupture process by a high stress drop patch of the same source. In this case, care has to be taken in determining the arrival times if the location of rupture initiation is sought, otherwise the location will be a statistical average of different locations within the source. It is helpful therefore to think of a seismic source as a spatial event with characteristic size of the order of a metre for a moment magnitude \( m_M = -4 \) event, up to a few hundred metres for an event of \( m_M > 4 \).

5.1 Location by arrival times and/or directions or azimuths

One can write the following arrival time equation for the \( j \)th seismic station that recorded the P or S wave:

\[
LOC_j(x) = t_j - t_0 - T_j(h) \quad (5.1)
\]

where:

- \( t_j \) the observed P or S wave arrival time at the \( j \)th station;
- \( t_0 \) the unknown origin time of the event;
- \( T_j(h) \) the unknown travel time of the P or S wave to the \( j \)th station;
- \( LOC_j(x) \) the residual i.e. the difference between the observed \([t_j]\) and the calculated \([t_0 + T_j(h)]\) arrival times.
Since equation (5.1) is nonlinear, one needs a well behaved system consisting of at least five equations to be able to find a unique solution. Due to the sparseness of and the large errors in the data and/or poor configuration of stations, the location problem can be unstable. A practical way to stabilize the solution is to eliminate the origin time, \( t_0 \), by centring, i.e. subtracting the average arrival time equation from the arrival time equations for each individual station. After centring and rescaling the residuals to metres and multiplying equation (5.1) by the average velocity of the P or S wave along the respective ray paths, it becomes

\[
LOC_j(h) = \bar{V}_j \left( t_j - \bar{t} - [T_j(h) - \bar{T}(h)] \right)
\]

(5.2)

where \( \bar{t} \) and \( \bar{T} \) are the average arrival time and average travel time respectively, and the origin time is found from \( t_0 = \bar{t} - \bar{T} \).

To constrain the location problem, one can add the direction of the wavefront, defined by the principal eigenvector of the covariance matrix of the initial P pulse recorded by the triaxial sensor at the \( j \)th station (Flynn, 1965; Mendecki, 1993). In this case, equation (5.2) becomes

\[
LOC_j(h) = \bar{V}_j \left( t_j - \bar{t} - [T_j(h) - \bar{T}(h)] \right) + Dis_j(h)
\]

(5.3)

where \( Dis_j(h) \) is the orthogonal distance between the current estimate of \( h \) and the direction, or vertical plane through the direction (azimuth), at the \( j \)th station. Directions will constrain the location problem only if the straight line is a fair approximation of the ray path between the source and the station, which is typically true for the nearby stations and/or if the medium can be considered homogeneous between the event and the station. The azimuth, rather than direction, should be used in the case of a layered geological structure with no significant lateral inhomogeneities. If the medium is very inhomogeneous, the travel times have to be calculated using a raytracing method rather than an arrival time equation based on a straight ray path.

In Fig. 5.1, an event recorded by the Vaal Reefs seismic network was located using the assumptions that the ray path is straight and that the medium is homogeneous. Also shown is the location based on the raytracing as discussed in Chapter 4. The velocity model used for the raytracing is shown in Fig. 4.8 and is based on the complex geology in this region. The difference in location is 4.2% of the average hypocentral distance (183 m) and, in this case, the straight ray path assumption is not adequate.

The classical way to solve a system of equations, each of type (5.3), is to linearize it with a Taylor expansion around a trial location and invert the resultant linear system, usually by a least squares method. This means that the