Nonlinearity and chaos are common features of many systems in nature, (Gutzwiller, 1991; Radu, 1995) though this has been acknowledged largely only during the past two decades. The phenomenon which goes under the name ‘chaos’ is an intrinsic feature of dynamics and comprises the large sensitivity to initial conditions of the evolution of a nonlinear dynamical system. In practice, it may happen that the response of a physical system varies greatly when small changes occur in the environment containing the physical system. This type of behaviour is regarded as chaotic.

Special methods and techniques have emerged from the study of systems that exhibit the phenomenon of chaos (Heiss et al., 1995). Their main merit is that they can sometimes offer alternative avenues to the traditional approach which consists of solving exactly the relevant equations of motion. Moreover, in the absence of a self-consistent model describing the dynamics of the process under study, it is not possible to gain any insight into the problem without the aid of more refined techniques. It is accepted that the occurrence of seismic events cannot currently be modelled with easy-to-use analytic equations. A study of the physics of seismicity is therefore expected to rely mostly on accurate data interpreted with the aid of nonlinear dynamics.

The seismic flow of rock is regarded as a highly complex process, intermittent in space and time, that provides for the occurrence of turbulence (Kagan, 1992; Chapter 10 of this book). In turn, turbulence is commonly associated with chaotic behaviour (Radu, 1995). It therefore appears that it is sensible enough to assume chaos may set in within the dynamics underlying the seismic flow of rock. In real systems, chaos and order coexist. By order we mean structures exhibiting a high degree of spatio-temporal regularity. Chaos and order both arise from the same type of nonlinear laws (Gutzwiller, 1991) and are often inseparable.

In order to make predictions about the occurrence of rockmass instabilities that may develop during the mining process, we need to manipulate the novel features of chaos theory. There is a connection between chaotic behaviour and instability. A system that is completely random but non-chaotic has a very stable configuration that may persist even in the presence of external perturbations. On the other hand, a system that is ordered may become unstable in the presence of external forces as its entropy has the opportunity to increase. The coupling between a regular system and external perturbations
is in general nonlinear and leads to chaotic behaviour. Rockmass subjected to external conditions (e.g. mining) is regarded as a nonlinear dissipative system far from equilibrium. In such a system, instability is the uncertainty about the path to be followed by the system in its phase space, a concept to which we turn next.

9.1 Phase space

When a nonlinear-type analysis is performed, the main object of study is the system's 'phase space'. The phase space of a physical system is the set of all possible states that the system can have. That collection of states can be spanned by a number of canonically independent variables (Gutzwiller, 1991). To render the notion of phase space clearer, we illustrate below the simple case of the one dimensional harmonic oscillator. This physical system is described by the following energy conservation relation:

\[ E = \frac{m\omega^2}{2} x^2 + \frac{p^2}{2m} \]  \hspace{1cm} (9.1)

where \( x \) is the position coordinate, \( p=mx \) is the momentum coordinate, \( m \) is the mass of the oscillator, \( \omega \) is the oscillation frequency and \( E \) is the energy of the oscillations. The variables that will span the phase space are \( x \) and \( p \). If we represent the above equation in the system's phase space we get an ellipse. All the possible states of the system (harmonic oscillator) lie on that particular ellipse (see Fig. 9.1). This configuration corresponds to a perfectly stable kind of motion.

**Fig. 9.1 Phase space configuration of the harmonic oscillator.**