NONINTEGRABLE, MULTIPLE SCALE FORMULATIONS

Mathematical Developments in continuum physics with boundary layers

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Abstract

After recalling the advantages and drawbacks of the vectorial and energy methods used in continuum physics, the need for a general formulation including coupled effects, irreversible processes and interfacial properties is placed in evidence. Such a formulation, recently developed by the author, required the establishment of new mathematical developments particularly at interfaces, generalizing thus the conventional theory of continuum physics well established in the bulk. The present paper deals with some mathematical procedures and complements a previous work where the basic physical principles were developed while the mathematical aspects were simply evoked.

1. Introduction

In dealing with coupled fields including mechanical and electromagnetic volume as well as interfacial properties, one meets a number of difficulties some of which require a reexamination of the foundations of continuum physics. Although two methods are available (the vectorial (newtonian) and energy (lagrangian) approaches) only the first of these is sufficiently general and may account for irreversible thermomechanics and electromagnetism [1] leading to differential equations in the bulk and to jump relations at an interface or to boundary conditions. As to the lagrangian formulation like the one mainly developed by Nelson in the last decades [2], it has two drawbacks. Firstly it applies only to reversible processes and secondly, unlike the vectorial approach it does not lead directly to jump relations and boundary conditions [2]. However, one of the main interests of the latter is that both mechanics and electromagnetism may be deduced in an elegant manner from a single scalar : Energy. Another interest is its accomodation to invariance principles such as invariance under rigid body motions, (also called objectivity principle or rotational invariance), that may be useful in some
complex structures where the vectorial approach cannot be applied safely [1, 3, 4, 5]. In the last decades and subsequent to the works of Germain [3] Maugin [4], the present author [5] and others a hybrid approach was developed (combination of the vectorial approach in so far as electromagnetism is concerned, with a weaker form of the lagrangian approach in view of including irreversible processes). In such an approach one can take benefit of the use of rotational invariance to model complex structures such as deformable piezoelectric semiconductors [5] and to account for irreversible processes which are essential in such materials. In spite of these practical interests, there exists a variety of circumstances where none of the above mentioned methods is applicable particularly when interfacial properties are taken into account. One of the simplest examples is that of magnetostatics. Indeed, the volume differential equations of mechanics are deduced from statements including closed two dimensional spaces, while those of electromagnetism require statements including closed two and one dimensional spaces. As to the surface differential equations they require additional postulates introduced by analogy to those present in the bulk except that one has to lower the dimension by one unit. This is completely possible in mechanics but partially possible in electromagnetism. These difficulties led to a number of ambiguities, partial treatments or inconsistencies [1, 5, 6, 7] placed in evidence in Ref. [8].

Recently, the author proposed a global solution applying to mechanics, electromagnetism, thermodynamics and their couplings, through the use of an energy formulation [8]. The latter is governed by three basic invariance principles, two of which are local or extrinsic and related to mechanics and electromagnetism (rotational invariance and gauge invariance). The third is global or intrinsic and accounts for the presence of discontinuities and interfaces avoiding thus the difficulties met with distribution products (scale-change invariance). A number of mathematical constructions were needed to the establishment of the above mentioned physical principles. Most of these constructions were simply evoked without any development. The author benefits of this occasion to present some procedures that lead to extended Green-Gauss and transport theorems which are essential in the framework of continuum physics and particularly in the above mentioned energy formulation.

The aim of the present paper is not to obtain fundamental mathematical innovations but rather to perform links between different contexts and procedures and to show possible extensions of some of them. This maybe helpful to the physicist dealing with interfaces (singular surfaces, lines and points). In sections 2 and 3, a natural extension from volumes to surfaces and lines is performed and a generalization of material volume, surface and line integrals of continuum mechanics is proposed. Section 4 is devoted to