1. Introduction

The questions on the interaction of a supersonic boundary layer with acoustic waves, which are considered in the present paper, were raised mainly in connection with the problem on the turbulence formation. At present the most complex problem on the prediction of the transition position in the boundary layer flows is related to the receptivity of these flows to the external effects. It appears that this problem was discussed in detail for the first time in [1], and during more than two subsequent decades many works were carried out on it. However, this problem has been studied more thoroughly both experimentally and theoretically for the subsonic flows. A review of the early works of the influence acoustic field effect on the transition from a laminar supersonic boundary layer to the turbulent has been given in [2]. The problems on the supersonic flow aeroacoustics were studied mainly within the framework of the investigations on the conditions for the onset of auto-oscillations and sound generation by the supersonic shear flows in the jets and mixing layers [3, 4]. The advanced approach based on the idea of the possibility of the mutual influence of the acoustic and hydrodynamic waves has been demonstrated in [5]. A relation between the acoustics problems and stability was shown thereby, to which an attention was drawn also in [10].

The first attempts at the investigation of the sound waves and supersonic boundary layer interaction on the basis of the stability theory were undertaken in [7, 8]. The problem on the excitation of unstable waves by sound was considered in [9].

The interaction of sound with a supersonic boundary layer was studied experimentally in [10], where the main results of the theory [8] were confirmed. The experiments of [11] using a controlled acoustic field have shown that the boundary layer receptivity to the acoustic disturbances depends on the location of the interaction region. It was, in particular, found that the intensity of the hydrodynamic waves engendered by the sound reaches its maximum values when the interaction region is located near the leading edge of the model, the lower branch of the neutral stability curve and the "sonic" branch of the neutral stability. The agreement between the
theoretical conclusions and the well-known experiments on the acoustic excitation of unstable waves is discussed in [12]. The generation of sound waves by a transitional boundary layer has been revealed experimentally in [13].

We analyze the state-of-the-art of the linear theory of the supersonic boundary layer receptivity to the acoustic disturbances on the basis materials obtained in the present work and on the available bibliographic data. The main attention is paid to the interaction of a longitudinal sound wave with the boundary layer.

2. General equations

Consider the interaction of a monochromatic wave with a boundary layer. The problem is solved in the approximation of the parallel flow in the boundary layer, which implies the independence of the main stream parameters of the longitudinal coordinate. To reduce the governing equations to a dimensionless form one introduces the reference length (the Blasius scale) 

$$\delta = \sqrt{x^* v_e / U_e}$$

and the time scale 

$$\tau = \delta U_e,$$

where \(x^*\) is the distance from the leading edge of a plate, \(v_e\) and \(U_e\) are the kinematic viscosity and velocity at the boundary layer edge, respectively. We will consider here the boundary layer on a flat impermeable plate, and the viscosity, velocity and temperature are related to the corresponding values at the external boundary of the boundary layer \(U_e, T_e, v_e\). A linear approximation is analyzed for the description of the nonstationary flow parameters, which is valid at small amplitudes of the sound wave.

The flow parameters of the incident sound wave are described by the vector function

$$Q^i(x, y, z, t) = \epsilon q_0 \exp \left[ i \left( \alpha x + \beta z + \gamma y - \omega t \right) \right],$$

where \(\epsilon\) is the wave intensity at the adopted normalization. Here \(x\) is directed along the main stream, and \(y\) is normal to the plate surface; \(z\) changes in the lateral direction (normal to \(x\) and \(y\)). The disturbances inside the boundary layer are described by the dependence

$$Q = q(y) \exp \left[ i \left( \alpha x + \beta z + \gamma y - \omega t \right) \right].$$

By using a conventional procedure of the linearization of the Navier-Stokes equations for the energy and continuity one can show that the components of the vector \(q(y)\) will satisfy a system of the eighth-order ordinary differential equations depending on the wave numbers \(\alpha, \beta, \gamma\), the frequency \(\omega\) and the main stream parameters \(U(y)\) and \(T(y)\). If we introduce the angle \(\chi = \arctan (\alpha / \beta)\) and go over from \(x, y, z\) to the new variables

$$x = x_1 \cos \chi - z_1 \sin \chi, \quad z = x_1 \sin \chi + z_1 \cos \chi,$$

then the disturbances will not depend on \(z\). The system of the eighth-order differential equations can be reduced under insignificant additional assumptions to a sixth-order system [2], which was obtained for the first time by Dunn and Lin [14]:

$$\frac{dq_i}{dy} = a_g q_j, \quad i, j = 1, \ldots, 6$$

(1)

We have used the following notations here: \(q_i\) and \(\alpha q_1\) are the disturbance amplitudes of the velocities in the \(x\)- and \(y\)-directions, \(\gamma q_1 \cos \chi q_6\), \(q_5\) are the disturbances of the pressures and the temperature; \(q_2\) and \(q_6\) are the derivatives with...