ABSTRACT. A definition of the degree to which a normalized convex fuzzy number is
greater or equal to another fuzzy number is proposed which satisfies the Ferrer's condition
and consequently the max-min transitivity.

It is based on the compensation of areas determined by the membership functions.

Using the fuzzy preference relation, one can handle inequality constraints, the related
conditions being free from any parametrization.

The particular case of $L-R$ fuzzy numbers is considered and it is proved that in this
case, comparison of areas is reduced to the comparison of upper and lower bounds of $\alpha$-cuts.

Two examples related to comparison of fuzzy numbers and fuzzy optimization are also
presented.

1. Introduction

We consider a set of normalized convex fuzzy numbers $A : \{a\}$ defined by the mem­
bership functions $\{\mu_a(x), x \in R\}$ with

$$\begin{cases}
\forall x \in R, \\
\mu_a(x) = 1 \\
\mu_a(y) \geq \mu_a(x) \land \mu_a(z), \quad \forall x \geq y \geq z \in R,
\end{cases}$$

where $\forall$ and $\land$ represent MAX and MIN operators.

The $\alpha$-level sets $I(a, \alpha) = \{x \in R : \mu_a(x) \geq \alpha\}$ are convex subsets of $R$ and there
is $m$ in $R$ such that $\mu_a(m) = 1$.

The lower and upper limits of any $\alpha$-level set $I(a, \alpha)$ are represented by $\inf_{x \in R} I(a, \alpha)$
and $\sup_{x \in R} I(a, \alpha)$. We suppose that both limits are finite.


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where

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These definitions give transitive relations \( \geq \) for \( L_\alpha, R_\alpha \), and are pessimistic compared with the definition obtained using the possibility index:

\[ a \geq b \iff \text{Poss.}(a \geq b) = \bigvee_{x>y} [\mu_a(x) \wedge \mu_b(y)] > 0 \]

which is however not necessarily transitive (the degree of possibility is shown to be negatively transitive: see Roubens and Vincke [6]).

We define a transitive compensatory \( \geq \) in the sense that values of \( \alpha \) for which \( \inf I(a, \alpha) + \sup I(a, \alpha) \geq \inf I(b, \alpha) + \sup I(b, \alpha) \) are compensating those values for which the inequality is not satisfied.

2. Definition

Let \( a, b \) be normalized convex fuzzy numbers and

\[ S_L(a \geq b) = \int_{U_1(a,b)} [\inf I(a, \alpha) - \inf I(b, \alpha)] d\alpha \]

\[ U_1(a,b) \text{ being the subset of } [0,1] : \{ \alpha \mid \inf I(a, \alpha) \geq \inf I(b, \alpha) \} \]

\[ S_R(a \geq b) = \int_{V_1(a,b)} [\sup I(a, \alpha) - \sup I(b, \alpha)] d\alpha \]

\[ V_1(a,b) \text{ being the subset of } [0,1] : \{ \alpha \mid \sup I(a, \alpha) \geq \sup I(b, \alpha) \} \]

The degree to which \( a \geq b \) is naturally considered as

\[ C(a \geq b) = C(a,b) = S_L(a \geq b) + S_R(a \geq b) - S_L(b \geq a) - S_R(b \geq a), \text{ if positive} \]

\[ = 0 \text{ otherwise} \]

The idea of compensation by integration can also be found in Baldwin and Guild [1] or Nakamura [3].

We will proof that the preference relation \( C \) is a fuzzy interval order.